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Quantitative techniques for simulating the impacts of a coal-fired power plant on the ground-water system of a river flood-plain wetland were developed and tested. Effects related to the construction and operation of the cooling lake and ashpit had the greatest impact. Ground-water flow system models were used to simulate ground-water flows before and after the cooling lake and ashpit were filled. The simulations and field data indicate that the cooling lake and ashpit altered local flow systems and increased ground-water discharge. Chemical changes in the ground-water system were minor. Contaminated ground water was confined to a small area near the ashpit. Thermal changes in the ground water are a major impact of the cooling lake. Changes in water temperature and levels have altered the vegetation of the wetland, a major ground-water discharge area. Ground-water temperatures near the cooling lake were monitored. A model was used to simulate the response of subsurface temperatures to seasonal changes in a lake and air temperatures. Long-term substrate temperature changes expected in the wetland were predicted. Using ground-water temperatures to estimate flow rates was investigated. Simulated temperature patterns agreed with field data, but were sensitive to the distribution of subsurface lithologies. It is predicted that by 1987 ground-water temperatures will be increased, resulting in an increase in ground-water flow.

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Wisconsin Power Plant Impact Study

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August 1980

IMPACTS OF COAL-FIRED POWER PLANTS ON LOCAL GROUND-WATER SYSTEMS

Wisconsin Power Plant Impact Study

by

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ENVIRONMENTAL RESEARCH LABORATORY-DULUTH
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FOREWORD

The U.S. Environmental Protection Agency was established to coordinate our country's efforts toward protecting and improving the environment. Research projects in a multitude of scientific and technical areas are necessary to monitor changes in the environment, to discover relationships within that environment, to determine health standards, and to eliminate potential hazards.

One such project, which the EPA is supporting through its Environmental Research Laboratory in Duluth, Minnesota, is the study "The Impacts of Coal-Fired Power Plants on the Environment." This interdisciplinary study, based at the Columbia Generating Station, near Portage, Wis., and involving investigators and experiments from many academic departments at the University of Wisconsin, is being carried out by the Environmental Monitoring and Data Acquisition Group of the Institute for Environmental Studies at the University of Wisconsin-Madison. Several utilities and state agencies are cooperating in the study: Wisconsin Power and Light Company, Madison Gas and Electric Company, Wisconsin Public Service Corporation, Wisconsin Public Service Commission, and Wisconsin Department of Natural Resources.

Reports from this study will be published as a series within the EPA Ecological Research Series. These reports will include topics related to chemical constituents, chemical transport mechanisms, biological effects, social and economic effects, and integration and synthesis.

This report describes the research undertaken by the Hydrogeology Subproject of the Columbia project. This research includes monitoring of ground-water flows and temperatures at the Columbia site, development of mathematical models for predicting such flows, and applications of the models to several problems.

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ABSTRACT

Quantitative techniques for simulating the impacts of a coal-fired power plant on the ground-water system of river flood-plain wetland in central Wisconsin were developed and tested by using field data collected at the site of the 500-MW Columbia Generating Station. The most important effects were those related to the construction and operation of the 200-ha cooling lake and the 28-ha ashpit.

Several two-dimensional vertically oriented steady-state models of the ground-water flow system were used to simulate ground-water flows before and after the filling of the cooling lake and ashpit. The simulations and supporting field evidence indicated that the creation of the cooling lake greatly altered the configuration of the local flow systems and increased the discharge of ground water to the wetland west of the site by a factor of 6.

Chemical changes in the ground-water system were minor. The plume of contaminated ground water originating from the ashpit was confined to a relatively small area near the ashpit. Thermal changes in the ground-water system are a major impact of the operation of the cooling lake inasmuch as the lake loses water to the ground-water system at a rate of 2×10^4 m^3 per day. The wetland, which is a major ground-water discharge area, has undergone a rapid and dramatic change in vegetation as a result of changes in both water temperature and water levels.

Ground-water temperatures in the vicinity of the cooling lake were monitored in detail for 1 1/2 yr. The response of subsurface temperatures temperature was simulated by means of a mathematical model, and predictions were made of the long-term changes expected in substrate temperatures in the wetland adjacent to the power plant. In addition, the use of ground-water estimate ground-water flow rates away from the cooling lake was investigated.

The model, which couples equations describing ground-water flow with those describing heat transport in the subsurface, was used to simulate the seasonal temperature fluctuation within seven cross sections oriented parallel to the direction of ground-water flow and downgradient from the cooling lake. Simulated temperature patterns agreed well with field data, but were very sensitive to the distribution of subsurface lithologies. The predictive simulations suggest that by 1987 temperatures at a depth of 0.6 m will not fall below 8°C within 200 m of the dike, and that peak temperatures near the dike will be 10-15°C above normal and will occur in October and November rather than in August. The increase in ground-water temperatures by 1987 will also result in a 24% increase in ground-water flow. The

subsurface stratigraphy of the site is such that major changes in near-surface temperatures will only occur within 340 m of the dikes, but, if the different, impacts could extend to much greater distances from the cooling-lake dikes.

This report was prepared with the cooperation of faculty and graduate students in the Department of Geology at the University of Wisconsin-Madison.

Most of the funding for the research reported here was provided by the U.S. Environmental Protection Agency, but funds were also granted by the University of Wisconsin-Madison, Wisconsin Power and Light Company, Madison Gas and Electric Company, Wisconsin Public Service Corporation, and Wisconsin Public Service Commission. This report was submitted in fulfillment of Grant No. R803971 by the Environmental Monitoring and Data Acquisition Group, Institute for Environmental Studies, University of Wisconsin-Madison under the partial sponsorship of the U.S. Environmental Protection Agency.

CONTENTS

Foreword iii
Abstract iv
Figures viii
Tables xii

1. Introduction 1
2. Conclusions and Recommendations 4
    Recommendations for siting future power plants 6
    Recommendations for future research 6
3. The Site of the Columbia Generating Station 8
4. The Impact of a Power Plant on the Ground-Water System of a Wetland 19
    Configuration of the ground-water flow system 19
    Wetland water levels 23
    Ground-water temperature 24
    Relationship between ground-water temperatures and flow rates 26
    Changes in water chemistry 26
5. Thermal Alteration of Ground Water Caused by Seepage from the Cooling Lake 30
    Mathematical model 30
    Results 35
6. Long-Term Temperature Changes in the Ground Water of the Wetland 41
    Simulation studies 41
    Results and discussion 43

References 51

Appendices

A. Data specifications for the water-flow model 57
B. Techniques for determining boundary conditions and parameters in the heat-flow model 61
C. A finite element program to simulate single-phase heat flow and conservative mass transport in a aquifer 73
D. Estimation of aquifer parameters by using subsurface temperature data 195

FIGURES

Number		Page
1	The site of the Columbia Generating Station during construction of the first unit (24 May 1971)	9
2	The site of the Columbia Generating Station after construction of both units (12 August 1977)	10
3	Main features of the site of the Columbia Generating Station . . .	11
4	Water flows and energy flows at the site of the Columbia Generating Station	12
5	Potentiometric surface contours in the area of the Columbia Generating Station	13
6	Representative stratigraphic cross sections of the subsurface at the site of the Columbia Generating Station	14
7	Topographic map of the site of the Columbia Generating Station . .	16
8	Peat thicknesses in the subsurface at the site of the Columbia Generating Station	18
9	Simulated head distribution in cross-secion A-A' of Figure 6 before filling of the Columbia cooling lake	20
10	Simulated head distribution in cross-section A-A' of Figure 6 after filling of the Columbia cooling lake	22
11	Water levels in the wetland west of the Columbia cooling lake before and after filling of the lake	24
12	Ground-water temperature variations in wells west of the Columbia cooling lake	25
13	Stiff diagrams of water in the cooling lake and ground water . . .	27
14	Stiff diagrams of water in the ashpit and in a well cased to 1.5 m below the surface and located 2 m west of the ashpit	28
15	Location of seven cross sections of the Columbia site for which ground-water temperature distributions were simulated	31

16 Observed temperatures in the subsurface west of the Columbia cooling lake and temperatures simulated by the mathematical model 36

17 Observed (solid line) and simulated (dashed line) ground-water temperature distributions in cross-section A-A' of Figure 15 . . 37

18 Observed (solid line) and simulated (dashed line) ground-water temperature distributions in cross-section B-B' of Figure 15 . . 38

19 Seasonal fluctuations of ground-water temperature in cross-section A-A' of Figure 15 at a depth of 4.5 m at distances of 2 m, 15 m, 50 m, and 84 m west of the cooling-lake dike 39

20 Simulated temperatures (solid lines) and observed temperatures (open circles) in cross-section A-A' of Figure 15 at a depth of 3.28 m at various distances west of the cooling lake 40

21 Cooling lake inlet temperatures for 1975-87 used in the simulations of long-term temperature change in the ground-water system 42

22 Predicted ground-water temperatures from 1975 to 1987 at a distance of 2 m west of the cooling-lake dike in cross-section A-A' of Figure 15 at a depth of 0.6 m 44

23 Pre-lake temperatures and simulated temperatures 2 m west of the cooling-lake dike in cross-section A-A' of Figure 15 at a depth of 0.6 m 45

24 Temperatures 150 m west of the cooling-lake dike in cross-section A-A' of Figure 15 at depths of 0.6 m and 3 m for the simulated period 1975-87 46

25 Temperatures 150 m west of the cooling-lake dike in cross-section A-A' of Figure 15 at a depth of 0.6 m 47

26 Temperatures in cross-section B-B' of Figure 15 at a depth of 0.6 m and at (a) 2 m west of the dike, (b) 50 m west of the dike, and (c) 200 m west of the cooling-lake dike 48-50

A-1 Location of monitoring wells at the Columbia site 58

B-1 Location of temperature sampling points at the Columbia Generating Station site 62

B-2 A simple circuit for measuring temperature 65

B-3 Temperature-resistance characteristics of Yellow Springs Series 400 thermistors and the change in resistance with a 0.1° change in temperature 67

B-4 Potted thermistor assembly showing the details of the thermistor mounts 67

B-5 Schematic of thermistor placement in subsurface wells at the Columbia Generating Station site 69

B-6 Schematic of digital bridge circuit used as a portable field meter for measuring ground-water temperatures 71

B-7 Details of (a) the needle probe and (b) the experimental arrangement to measure thermal conductivities of unconsolidated materials 72

B-8 Thermal conductivities of three ground-water samples from the Columbia Generating Station site which were analyzed with the needle probe 72

C-1 The basic procedure for linking the ground-water flow and the transport equations of the model 73

C-2 Examples of refinement of a finite element grid 81

C-3 Examples of numbering of nodes in a structure 93

C-4 Cartesian orientation of finite element grid 94

C-5 Typical elements of finite element grid and correct method of numbering the elements 95

C-6 Numbering of nodes and elements in rectangular grid 95

C-7 Two variants of the rectangular grid that can be generated 96

C-8 Numbering of sides of an element 97

C-9 Finite element grids used to discretize a linear heat transport problem 104

C-10 Input data used to model one-dimensional heat transport with all linear elements 105

C-11 Program output for one-dimensional heat transport problem with linear elements 107

C-12 Data deck used to model one-dimensional heat transport with mixed elements 111

C-13 Program output for the one-dimensional heat transport problem with mixed elements 113

C-14 Analytical solutions (solid lines) and numerical solutions (dots) for the linear heat transport problem for t = 25, 50, and 75 days 118

C-12 Areal view of the Mohawk River Valley showing location of the Schenectady and Rotterdam well fields 121

C-16 Cross-sectional view of the Mohawk River alluvial aquifer along section A-A' of Figure C-15 121

C-17 Grid used to discretize the Mohawk River problem 122

C-18 Data deck used to model heat flow in the Mohawk River alluvial aquifer 123

C-19 Program output for Mohawk River problem 125

C-20 Schematic cross section of the Columbia Generating Station site along an east-west line 131

C-21 The grid used to discretize the cross section simulated at the Columbia Generating Station site 131

C-22 Data deck used to model heat flow at the Columbia Generating Station site 132

C-23 Program output for Columbia Generating Station problem 134

C-24 Grid used to discretize the aquifer simulated in the heat pump problem 138

C-25 Data deck used to model the heat pump simulation with no regional ground-water flow 139

C-26 Program output for the heat pump problem 141

C-27 Program flow chart 147

D-1 Temperatures (°C) in a section of the marsh adjacent to the cooling lake at the Columbia Generating Station site on 7 October 1977 200

D-2 Temperatures in a 60-m portion of cross-section B-B' of Figure 6 adjacent to the drainage ditch east of the cooling lake on 9 June 1977 and 24 October 1977 201

TABLES

Number Page

1 Horizontal Hydraulic Conductivities for the Lithologies in the Subsurface of the Columbia Generating Station 15

2 Selected Chemical Concentrations in Ground Water Near the Ashpit of the Columbia Generating Station 29

3 Parameters Used in the Simulations for the Model Describing Thermal Alteration of Ground-water 35

B-1 Temperature Sampling Points: Locations, Depths, Methods, and Frequency of Readings 63

B-2 Heat Capacities of the Common Components of Unconsolidated Glacial Materials 72

C-1 Parameters for Both Mass and Heat Transport Problems 76

C-2 Parameters, Initial Conditions, and Boundary Conditions Used for the Heat-Flow Equation in the Linear Heat Transport Problem . . 106

C-3 Analytical Solution at t = 50 Days for the Problem Posed in Figure C-9 and Finite Element Numerical Solutions at t = 50 Days for Two Grid Configuations and Two Types of Boundary Conditions 117

C-4 Parameters, Initial Conditions, and Boundary Conditions Used for the Simulation of Temperatures in the Mohawk River Alluvial Aquifer 120

C-5 Parameters, Initial Conditions, and Boundary Conditions Used in the Columbia Generating Station Problem 130

SECTION 1

INTRODUCTION

Large industrial facilities are common features of the American landscape, but knowledge of the mechanisms by which they change natural systems is inadequate. The Hydrogeology Subproject of the interdisciplinary research project "The Impacts of Coal-Fired Power Plants on the Environment" quantified the effects of construction and operation of the Columbia Generating Station on the hydrogeologic system of an adjacent wetland. The investigators developed techniques for simulating the observed changes and for predicting potential impacts at other sites. Impacts on the ground-water system of the wetland resulted from the construction of a 200-ha cooling lake and a 28-ha ashpit. Specifically, this study assesses the alteration of three characteristics of the ground-water system: (1) the quantity of ground-water flow from the cooling lake and the ashpit into the wetland and surface-water levels in the wetland, (2) the quality of ground water and surface water as expressed by concentrations of the common cations and anions, and (3) the temperatures of ground water and surface water at the site.

Data on ground-water temperatures were collected from 1971, 4 yr before the operation of the Columbia Generating Station, through 1977. The basic monitoring network consisted of 100 small-diameter wells and a subsurface network of 64 temperature monitoring points.

During the initial phase of the study the ground-water flow systems in the vicinity of the generating station were delineated and the way in which the flow systems were altered by the construction and operation of a cooling lake and ashpit were documented. A finite difference model was used to simulate the alterations in the configuration of the ground-water flow system. Analyses of changes in the chemical quality and temperature of ground water were descriptive during this phase of the study.

The second phase of the study dealt with the monitoring and simulation of the transfer of one byproduct of the generating process, specifically heat, away from the site via the ground-water system. The study of the transfer of heat was chosen rather than the transfer of chemical byproducts because: (1) temperatures can be monitored inexpensively and rapidly at a large number of points, (2) the processes of heat transfer are well understood, and (3) the initial phase of the study had shown that alterations in the temperature of ground water were large, whereas alterations in the chemical characteristics of ground water were small. The development of the capability to simulate the transfer of heat is a prerequisite to studying the flow of a chemical away from the site. The processes determining the chemical composition of ground water are poorly understood, and it is difficult to obtain representative ground-water samples.

Data on ground-water temperatures were collected for an 18-mo period during 1976-77 in the vicinity of the cooling lake of the Columbia Generating Station. The finite element method, using isoparametric quadrilateral elements, was used to solve the partial differential equations describing ground-water flow and heat transport in the subsurface. The model was tested by comparing the observed ground-water temperatures in the vicinity of the cooling lake with predicted temperatures. The simulated temperatures were in close agreement with the observed temperatures.

The model was then used to simulate the long-term (12-yr) effects of seepage from the cooling lake on ground-water temperatures near the lake. The temperature data collected at the Columbia Generating Station site and the model were also used to refine estimates of ground-water flow from the cooling lake calculated by using potentiometric data. A third application of the model was the simulation of the impacts on ground-water temperatures on the use of heat pumps for residential heating and cooling (Andrews 1978).

Although ground-water systems have been explored for possible energy storage or as a source of energy, the published literature contains no previous work documenting the effects of a power plant on a ground-water system. The possibility of temporary storage of energy in the form of heated water in aquifers has been explored by Meyer and Todd (1973), Hausz and Meyer (1975), Kley and Nieskens (1975), Molz et al. (1976), Tsang et al. (1976), and Werner and Kley (1977). Gass and Lehr (1977) advocated the use of ground water itself as an energy source. Gringarten and Sauty (1975), Intercomp Resource Development and Engineering (1976), and Tsang et al. (1976) developed models of the transport of heat in ground-water systems to study the feasibility of temporary storage of heated water in aquifers. Mercer et al. (1975) modeled the movement of heat and water in a hydrothermal system.

Alterations in the ground-water system induced by activities at a power-plant site can bring about significant changes in nearby flora, fauna, and surface waters. For example, in a wetland environment, here defined as an area where the water table is at or near the surface at all times, alterations in ground-water quality and ground-water discharge rates have been shown to alter wetland ecosystems (Bay 1967, Dix and Smeins 1967, Walker and Coupland 1968, Millar 1973). In addition, Vadas et al. (1976) and Gibbons (1976) found that changes in wetland water temperatures led to changes in wetland ecosystems.

Boulter (1972) studied the extent of water-table drawdown after a wetland is ditched. But no one has investigated the changes that occur in water levels, water quality, or water and substrate temperatures in a wetland when a ground-water system discharging into a wetland is altered. In the wetland adjacent to the Columbia Generating Station, wetland vegetation changed quickly and markedly following changes in water temperatures, water levels, and water flows caused by the presence of the cooling lake (Bedford 1977). During the first 2 yr of operation of the Columbia Generating Station, a community previously dominated by sedge meadow species was replaced by emergent aquatic species and annuals (Bedford 1978).

This report deals with the hydrogeologic environment of the site of the Columbia Generating Station and the major effects of power-plant construction and operation; the impacts of the generating station on the ground-water system; the simulation of heat flow in the subsurface near the site; the expected long-term changes in ground water at the site; and the major conclusions and recommendations of the study. The appendices contain a description of the field methods used in the study (appendix A and appendix B), the computer program for solving the model of heat and water flow in shallow ground-water systems (appendix C), and a discussion of the use of the heat- and water-flow model to refine estimates of flow using temperature data (appendix D). An additional application of the model is reported by Andrews (1978).

SECTION 2

CONCLUSIONS AND RECOMMENDATIONS

The cooling lake at the Columbia Generating Station has dramatically altered the water supply to the marsh west of the lake. Before the filling of the lake, ground water, the major source of water to this area, discharged at a rate of less than 0.03 m^3/s annually. The discharge rate increased by a factor of 6 after the filling of the cooling lake. The increased water supply to the marsh has raised surface-water levels in the sedge meadow approximately 10 cm above pre-lake levels, and the water levels now have much less seasonal variability.

The changes in ground-water patterns have been confined to the area between the station's cooling lake and ashpit and the Wisconsin River. To the east of the cooling lake, a drainage ditch limits the effects of the lake on the ground-water system to a narrow band east of the lake, except in the northeast corner of the lake where there is no ditch. In this area the water table has risen several feet, but flow in this area is small.

Thermal patterns in the marsh have been significantly altered by the cooling lake. Temperatures of the ground water that discharges in the marsh now average several degrees above prior average ground-water temperatures, and temperatures in the marsh are out of phase with seasonal air-temperature patterns. Much of the sedge meadow no longer freezes over in winter, and temperatures just below the surface are as high as 25° C prolong the growing season. Winter temperatures as high as 21° C and summer temperatures as low as 8° C were observed. In some areas the peak ground-water temperature occurred during the winter months. Changes in vegetation within the thermally altered zone were documented by Willard et al. (1976) and are thought to occur in response to changes in ground-water temperatures.

As with changes in ground-water flows, the thermal alteration of ground water was confined to the area west of the cooling lake. Moreover, significant changes in temperatures extended only to about 100 m west from the dike. The seasonal maximum ground-water temperature at any depth is a function of distance from the cooling lake and the distribution of subsurface lithologies.

The movement of heat in the subsurface was simulated by using the finite element method to approximate the differential equations for water and heat flow. The simulated temperature patterns agreed well with field data, but were very sensitive to the distribution of subsurface lithologies. Detailed stratigraphic information was necessary to obtain reliable results. The problem of simulating heat flow in the subsurface is further complicated by a poor understanding of how to estimate dispersivity. Results presented in this study show that the amplitude of the seasonal temperature wave is best simulated when small dispersivities are used.

Long-term simulations of thermal alterations in the ground-water system of the marsh indicate that the cooling lake will significantly alter substrate temperatures but only within 350 m of the cooling lake dike. By 1987 peak temperatures near the dike were elevated 10-15° C above normal levels and lagged behind seasonal temperature changes by 1-2 months. At 150 m from the dike, peak temperatures at a depth of 0.6 m were only elevated 2.3° C above normal levels in the summer, but winter temperatures were elevated 6-10° C above normal levels. Temperatures within the wetland varied spatially, but trends were expected to be similar to those simulated in the two cross sections modeled. Seepage rates will increase 20% as the result of the increase in temperature of seepage waters.

The simulations indicate that winter temperatures will be warm enough to prevent formation of an ice cover in almost the entire marsh between the cooling lake and the Wisconsin River. Near the dikes a relatively warm microclimate will exist during most of the winter, since the temperature of the discharging water will be above 20° C during the early winter.

The potential for significant thermal alteration of surface-water bodies located in ground-water discharge zones is great. The temperature of the ground water may rise considerably near a cooling lake, even though the total heat flux is small. At the site of the Columbia Generating Station 40% of the water pumped into the cooling lake seeps into the ground-water system, yet less than 2% of the waste heat load is discharged to the ground-water reservoir. The discharge rate of water from the generating station to the cooling lake is 1×10^6 m^3 per day, and the temperature of the surface water is increased 10-15° C. However, because the seepage rate is only 2×10^4 m^3 per day, much of the heat load is dissipated through evaporation. The percentage of the heat load that leaves a cooling lake via seepage to the ground-water system is unlikely to be greater at other power-plant sites.

Seepage from the ashpit averages 0.3-0.6 m^3/s. Surface water in the wetland within 50 m of the ashpit has been contaminated by seepage through the ashpit dikes, and total dissolved solids in the surface waters have increased over fivefold above background levels in this area. However, significant chemical degradation of ground water, judged by the concentration of the common cations and anions, has not occurred in the vicinity of the ashpit.

A plume of contaminated ground water is slowly moving eastward from the coal pile. Three years after coal was piled in the area, a plume of water with a sulfate concentration greater than 1,000 mg/liter extended to a depth of greater than 30 m and outward from the limits of the coal pile more than 100 m.

RECOMMENDATIONS FOR SITING FUTURE POWER PLANTS

All new power plants should be required to gauge accurately (within 1%) all water flows into and out of the plant and the plant site. In addition, precipitation into and evapotranspiration from all bodies of open water, such as ashpits and and cooling lakes, should be monitored. Seepage from ashpits and cooling lakes cannot be determined accurately by direct methods; instead, seepage should be calculated indirectly by a mass balance approach. Data on water flows, precipitation, and evapotranspiration are required for calculating a mass balance. Control rooms of power plants are now designed to measure accurately mass flows within the generating station, and therefore water flows between the environment and the plant can also be carefully monitored.

Coal-storage areas should be designed so that water infiltrating the coal pile cannot reach the water table. Since crushed coal is highly permeable and leachable, an open coal pile allows most of the annual precipitation to pass through the pile, which creates water similiar to acid mine drainage. Unless infiltration is controlled or the infiltrating water is captured and treated, a volume of contaminated water equal to the product of the annual precipitation and the area of the coal pile will flow into the subsurface.

Estimations of impacts of generating stations should recognize that significant thermal alterations of nearby surface waters can occur even when a closed-cycle cooling lake is used.

Dikes of all ashpits and cooling lakes should be lined with several inches of a relatively impermeable clay material. Seepage can be controlled, and sloppy designs should not be tolerated. More seepage can usually be allowed from a cooling lake than from an ashpit, since cooling-lake water is usually less threatening to the environment.

Simulation techniques should be used to evaluate potential seepage from ashpits, cooling lakes, and coal-storage areas in the design stage of these structures. Designs should minimize ground water seepage and contamination. Competent geologic consulting firms should be retained to conduct these studies.

Although seepage from a cooling lake may contribute to alterations in an adjacent wetland ecosystem, the high leakage rates are not undesirable from a lake-management standpoint. Seepage to the ground-water system prevents an increase in total dissolved solids in the lake. Such an increase would necessitate periodic flushing of the lake and the consequent release of saline water.

RECOMMENDATIONS FOR FUTURE RESEARCH

More field data are needed on the distribution of temperature or conservative chemical characteristics, or both, in small ground-water systems subjected to stresses that are altering these characteristics from background levels. The flow of mass and energy in ground-water systems is much more difficult to quantify than the flow of water, mainly because the former is sensitive to the hydraulic conductivity distribution. The development of quantitative techniques to date has been hampered by the lack of field data to verify the models. As demonstrated in this study, it may be more fruitful to monitor temperature changes than chemical changes in a ground-water system if the goal is to determine the system's capability to transport mass and energy. Temperature can be inexpensively and rapidly monitored at a large number of points, which is not the case for chemical characteristics.

Existing ground-water models will be adequate in most cases for quantifying impacts of power plants on hydrogeologic systems. The best existing models for these purposes are those devised by the U.S. Geological Survey. Future research should not be directed towards developing new models for these purposes, unless the investigator can show clearly that existing models are deficient.

More study is needed of the factors controlling dispersivity, and the appropriateness of the concept should be evaluated.

An evaluation is needed of the prediction ability of models used when historical data are not available. The magnitude of error associated with predictions should be quantified. It may be that transport models should not be used for predictions if historical data are not available for calibration.

A theoretical evaluation of the validity of using either chemical or energy distribution in the subsurface to determine hydraulic conductivity distributions is needed. The question to be asked is whether or not the inverse approach can produce useful approximations in a problem that is more complex than those investigated by Bredehoeft and Papadopulos (1965).

SECTION 3

THE SITE OF THE COLUMBIA GENERATING STATION

The Columbia Generating Station is a 1,000-MW coal-fired electric-power-generating complex located on the flood plain of the Wisconsin River 5 miles south of Portage, Wis. Construction began in 1971, and the two 500-MW units began operation in May 1975 and May 1978. The 1,620-ha site on which the facility was constructed consisted of an extensive marsh, mainly sedge meadows, and wetland forests with sandy upland knolls (Figure 1). Construction activities have dramatically altered the site. A 200-ha cooling lake, a 28-ha ashpit, a 16-ha coal pile, and the generating station itself are now the dominate features of the site (Figure 2, Figure 3).

The power plant dynamically interacts with the environment. On an average day the first unit alone burns 2,000 tons of coal, producing 385 MW of electricity, 1,000 MW of waste heat, and 552 tons of coal residue. This rate of coal burning is approximately 38 kg/s, which represents over 1,000 MW of power of which only about 360 MW are converted to electricity. The cooling lake and the ashpit are the direct recipients of most of the byproducts. The thermal and chemical characteristics of the byproducts differ markedly from the thermal and chemical characteristics of water in the adjacent wetland environments (Figure 4).

The site of the Columbia Generating Station is located in a regional discharge area (Figure 5). The discharge area includes the wetlands on the site of the power plant, the wetlands to the east of the site, which are used for mint farming, and extensive wetlands along the lower reaches of Rocky Run Creek. The marsh on which the site is located occupies a former river channel.

The geology beneath the site of the Columbia Generating Station consists of irregularly eroded Upper Cambrian quartz sandstone overlain by sediments from glacial lake bottoms and drift deposits of the Green Bay lobe of the Wisconsin ice sheet. The bedrock is composed of Upper Cambrian sandstones and Precambrian granites which occur 125 m below the surface. The surface is covered by a layer of peat 1.5 m thick, which overlies a thin layer of organic clay and silt. The peat and clay are in turn underlain by alluvial sands with clay lenses. Representative stratigraphic cross sections are shown in Figure 6. The hydraulic conductivity of the clay is an order of magnitude less than the peat and thus controls the rate of ground-water discharge (Table 1). Vertical hydraulic conductivities were estimated to be 5 to 20 times less than horizontal hydraulic conductivities.

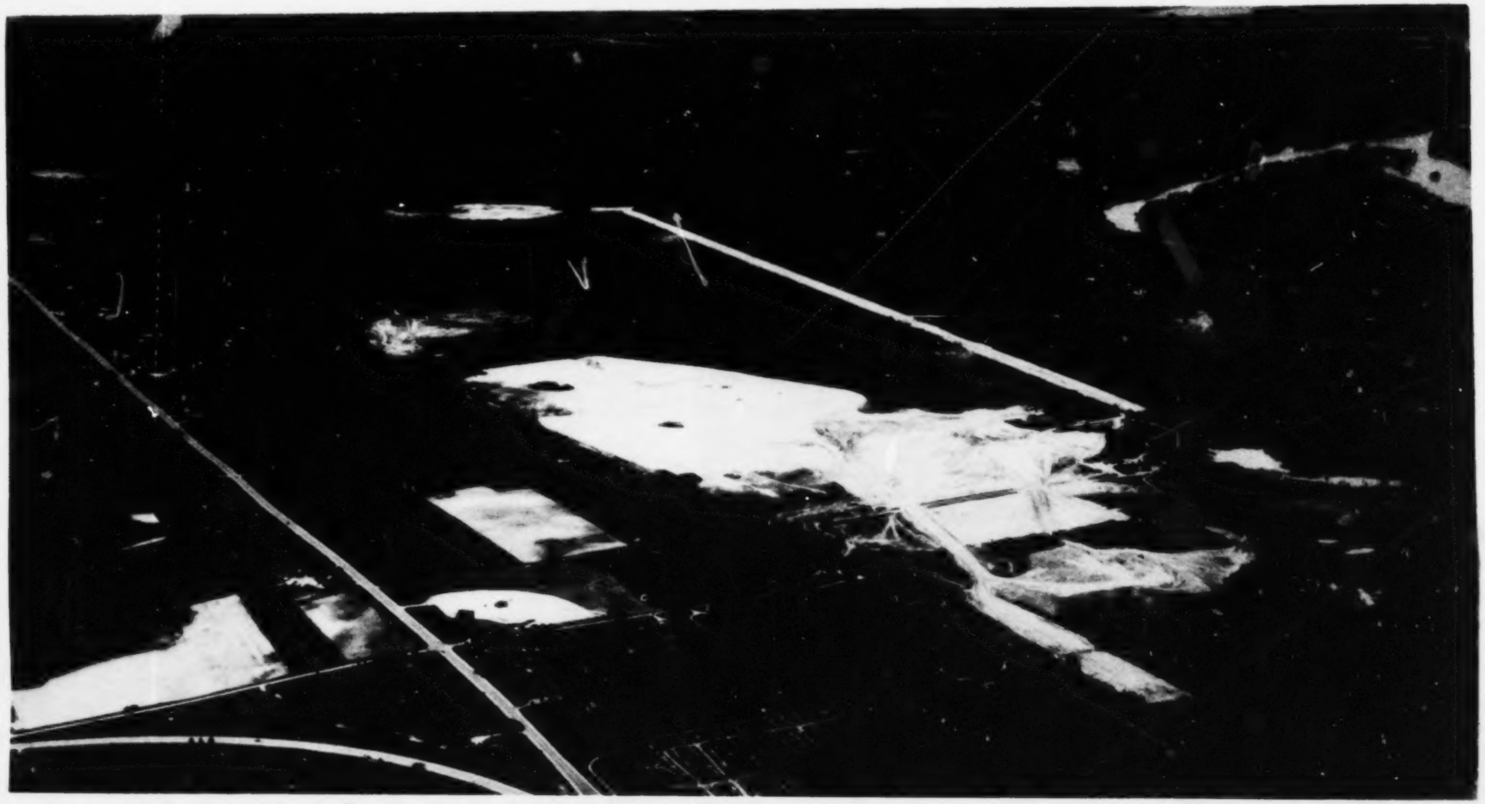

Figure 1. The site of the Columbia Generating Station during construction of the first unit (24 May 1971).

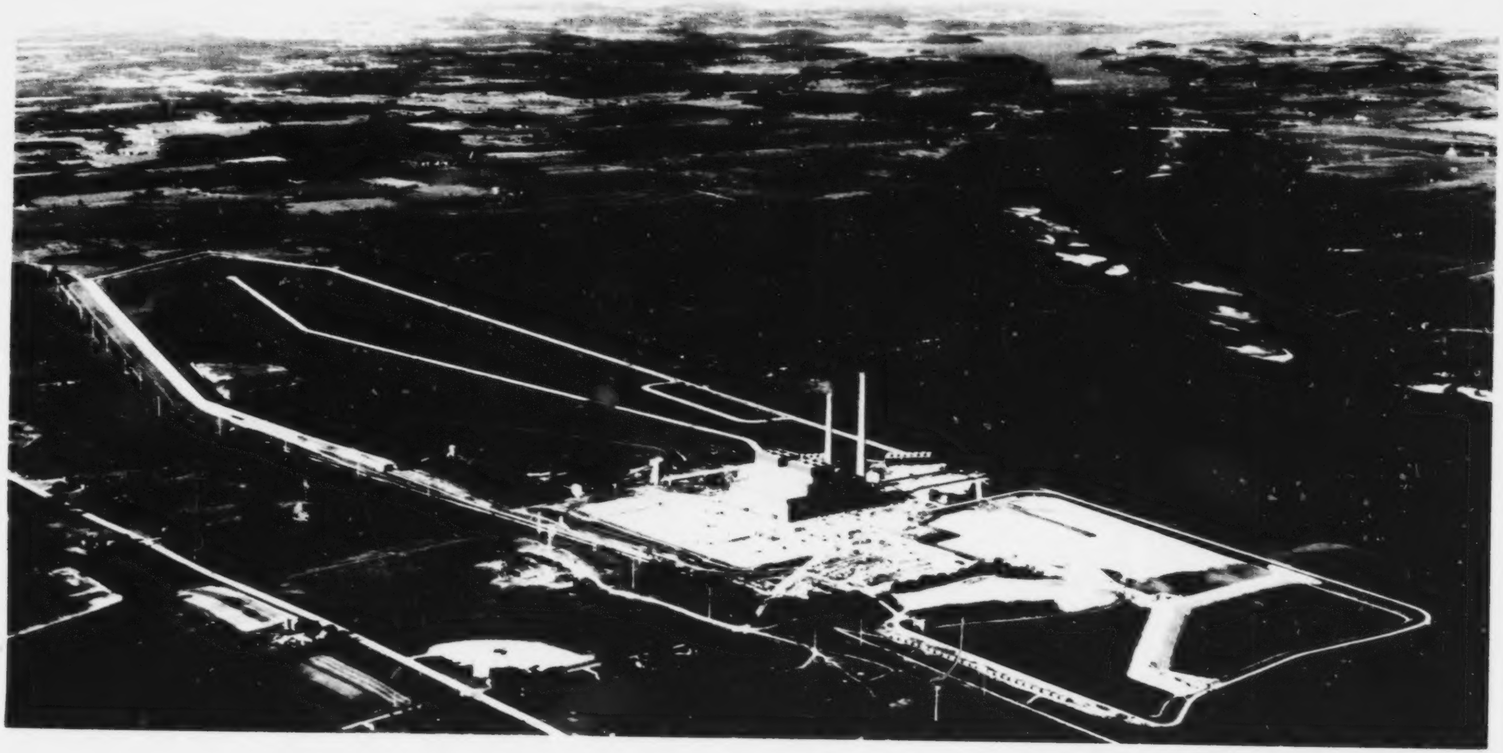

Figure 2. The site of the Columbia Generating STation after construction of both units (12 August 1977).

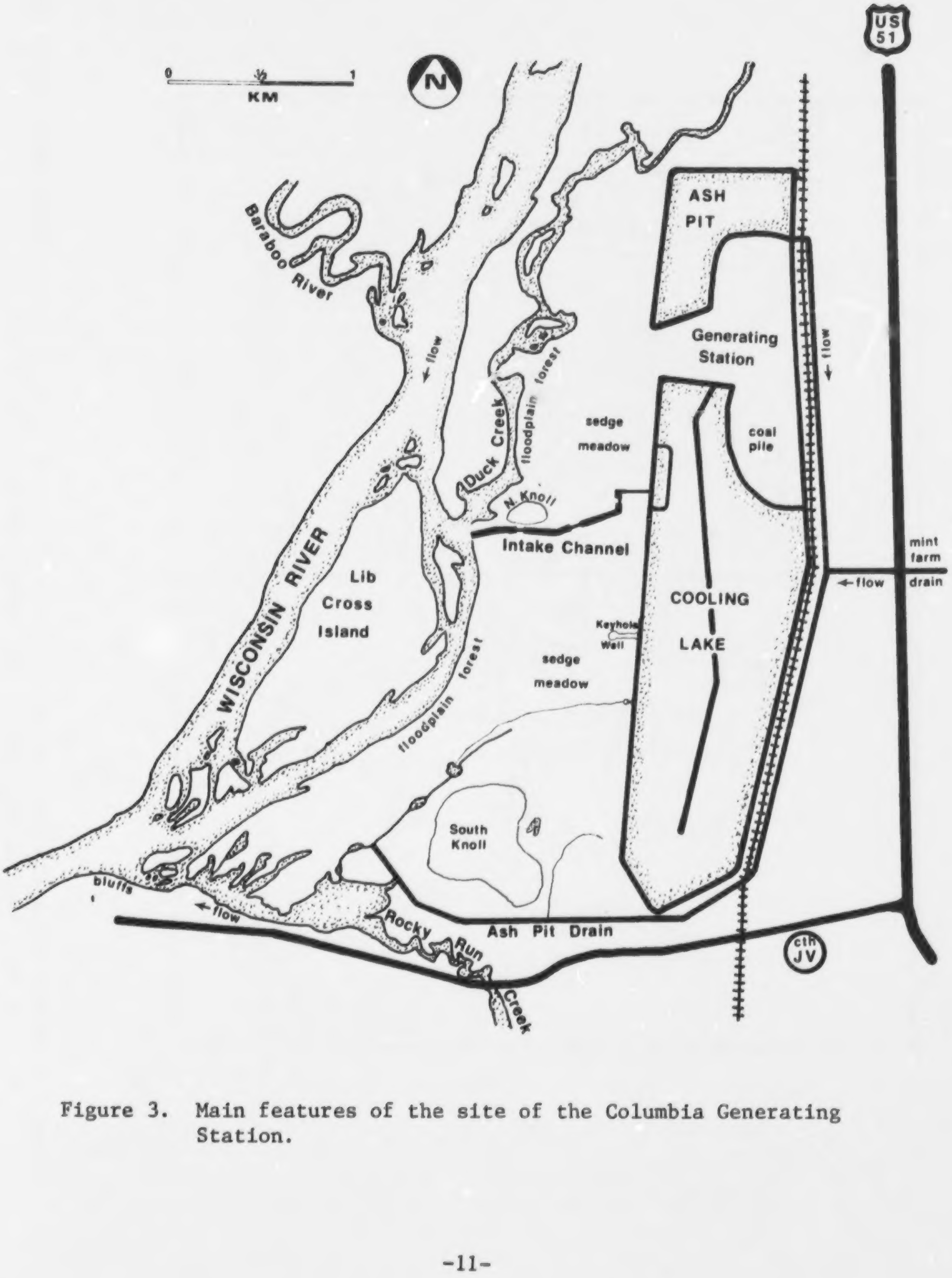
0
½
1
KM
N
US 51
ASH PIT
Baraboo River
flow
Generating Station
flow
Duck Creek
floodplain forest
sedge meadow
coal pile
N. Knoll
Intake Channel
mint farm drain
flow
WISCONSIN RIVER
Lib Cross Island
COOLING LAKE
Keyhole Well
sedge meadow
floodplain forest
South Knoll
bluffs
flow
Rocky Run Creek
Ash Pit Drain
cth JV

Figure 3. Main features of the site of the Columbia Generating Station.

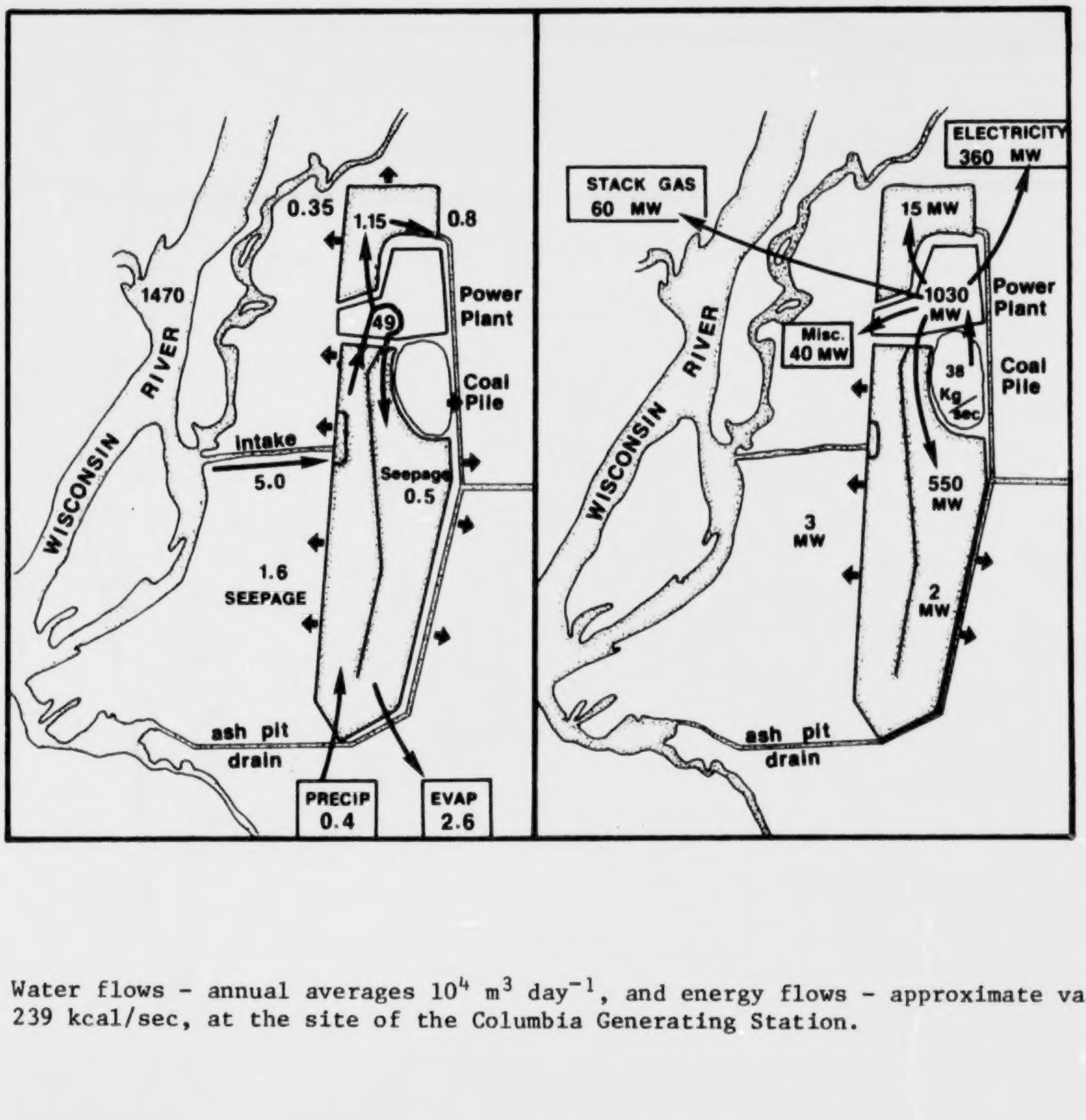
WISCONSIN RIVER
1470
0.35
1.15
0.8
Power Plant
49
Coal Pile
intake
5.0
Seepage
0.5
1.6
SEEPAGE
ash pit drain
PRECIP
0.4
EVAP
2.6
STACK GAS
60 MW
ELECTRICITY
360 MW
15 MW
1030 MW
Misc.
40 MW
38 Kg/sec
550 MW
3 MW
2 MW

Figure 4. Water flows - annual averages 10^4 m^3 day^{-1}, and energy flows - approximate values 1MW = 239 kcal/sec, at the site of the Columbia Generating Station.

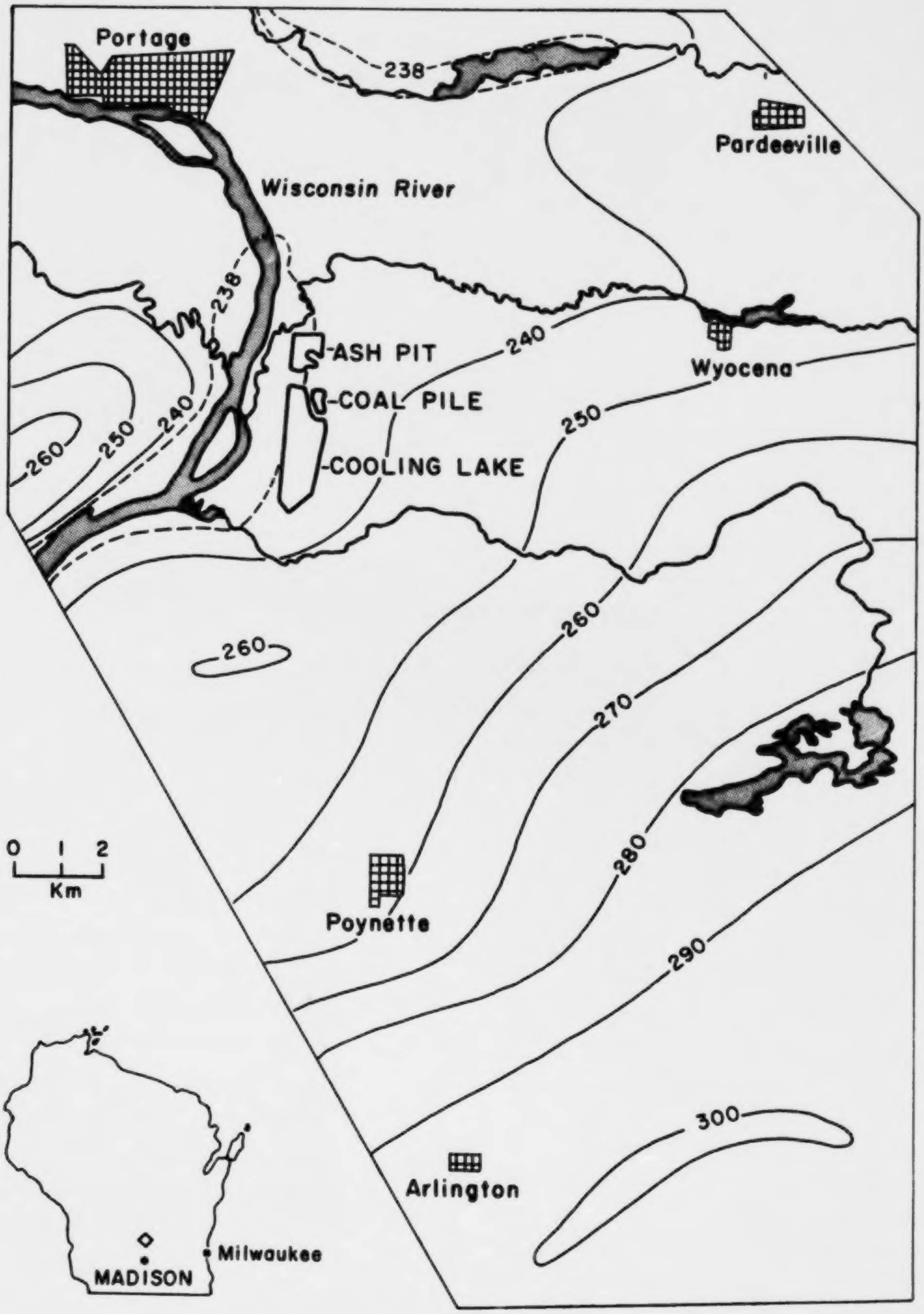
Portage
238
Pardeeville
Wisconsin River
238
ASH PIT
240
Wyocena
COAL PILE
250
260
250
240
COOLING LAKE
260
260
270
0 1 2
Km
280
Poynette
290
300
Arlington
MADISON
Milwaukee

Figure 5. Potentiometric surface contours in the Cambrian sandstone (meters above mean sea level) in the area of the Columbia Generating Station. Location is shown by diamond on insert map of the State of Wisconsin.

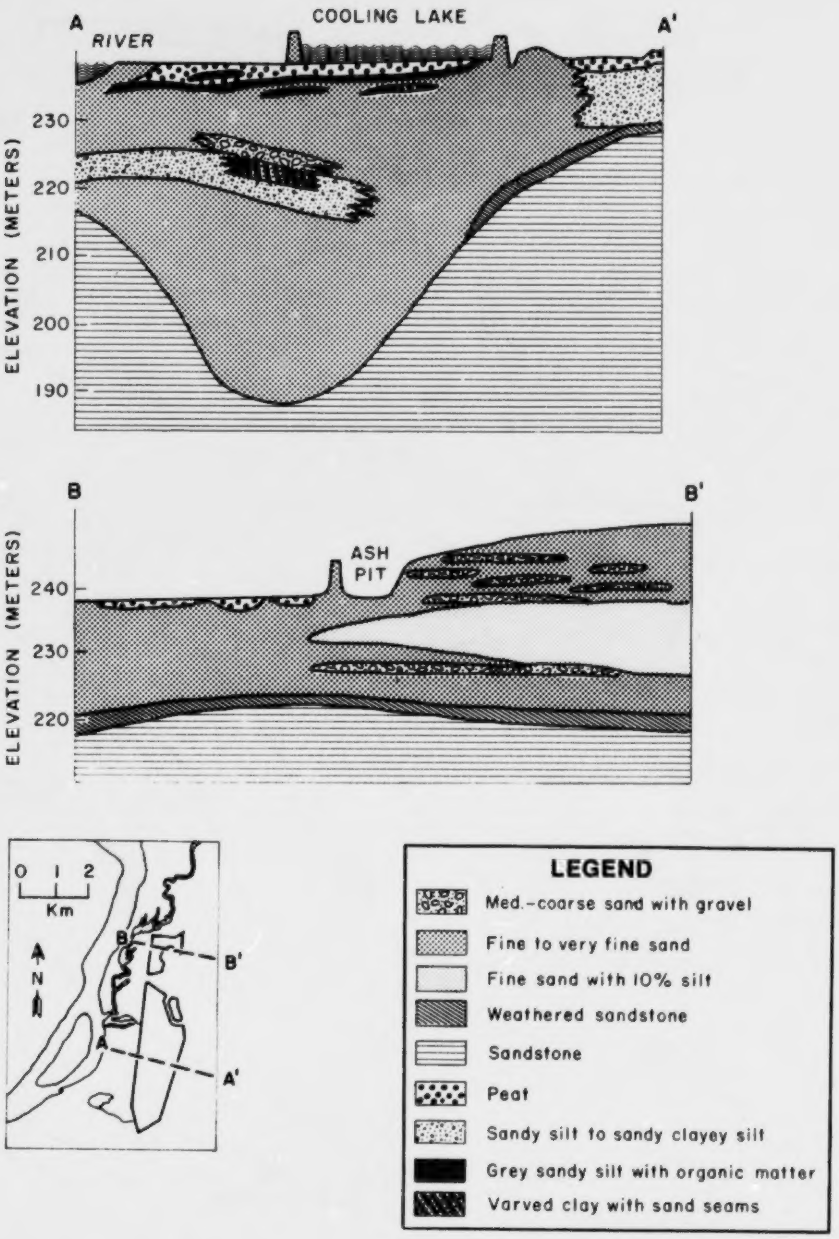
A
RIVER
COOLING LAKE
A'
ELEVATION (METERS)
230
220
210
200
190
B
B'
ASH PIT
ELEVATION (METERS)
240
230
220
0 1 2
Km
N
B
B'
A
A'
LEGEND
Med.-coarse sand with gravel
Fine to very fine sand
Fine sand with 10% silt
Weathered sandstone
Sandstone
Peat
Sandy silt to sandy clayey silt
Grey sandy silt with organic matter
Varved clay with sand seams

Figure 6. Representative stratigraphic cross sections of the subsurface at the site of the Columbia Generating Station. Insert shows locations of sections at the site.

TABLE 1. HORIZONTAL HYDRAULIC CONDUCTIVITIES FOR THE LITHOLOGIES IN THE SUBSURFACE OF THE COLUMBIA GENERATING STATION

Lithology	Horizontal hydraulic conductivity (m/day)[a]
Medium-coarse sand with gravel	30
Fine to very fine sand	10
Fine sand with 10% silt	6
Weathered sandstone	8
Sandstone	3--4
Peat	0.4--3
Sandy silt to sandy clayey silt	0.4
Gray sandy silt with organic matter	0.04
Varved clay with sand seams	0.02--0.04

[a] Conductivities were determined from a pump test, slug tests, laboratory permeameter tests, and estimates based on lithology and grain-size analyses.

The 1,900-ha site ranges in elevation from 237 to 248 m above mean sea level. The higher areas are well drained since the underlying soil is fine to medium sand. The elevation of most of the site, however, is less than 239 m, and these low areas are usually wet throughout the year because of ground-water discharge (Figure 7).

A well-developed ground-water flow system exists in the Cambrian sandstone. The discharge areas of the aquifer include the site, the wetlands west of the cooling lake, the wetlands east of the lake, and extensive wetlands along the lower reaches of Rocky Run Creek (Figure 3). The sandstone aquifer has an areal extent of approximately 200 km^2 extending from the site to the southeast (Figure 5). Discharge from this aquifer is nearly constant all year and averages about 0.013 m^3/s per kilometer perpendicular to the direction of flow.

The low areas also receive ground-water recharge from the upland areas located on the site and adjacent to the site. These upland areas act as recharge areas during precipitation events, when the infiltrating water flows toward the low areas. Travel time for water from the upland areas to the low

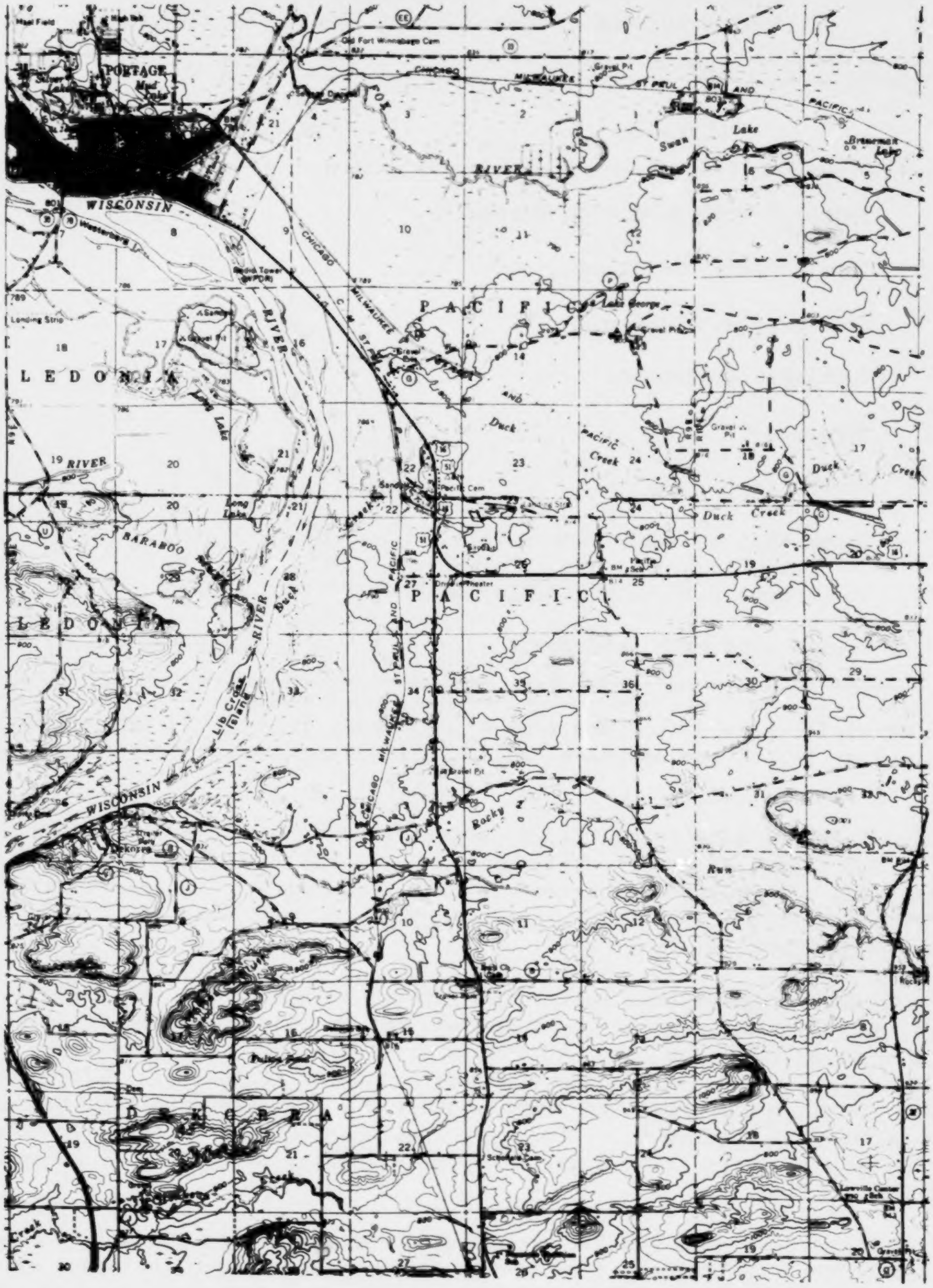
PORTAGE
WISCONSIN
FOX
RIVER
Swan
Lake
P A C I F I C
L E D O N I A
Duck
Creek
BARABOO
RIVER
WISCONSIN
Rocky
Run
P A C I F I C
L E D O N I A
D E K O R R A

Figure 7. Topographic map of the site of the Columbia Generating Station.

areas is usually less than 3 months, but flow rates are seasonally variable. Before construction of the cooling lake, approximately 50% of the ground water discharging in the low areas in the spring originated from the upland areas on and near the site; during the summer the percentage dropped to less than 5%.

The ground-water inflow rates to the low areas are greatest where the peat deposits are thinnest (Figure 8), namely in areas recently reworked by fluvial processes. The peat deposits are generally less than 2 m thick west of the railroad track; east of the railroad track, kettles in the outwash and ice-contact drift have been filled in by organic deposits as thick as 10 m. North of the ashpit alluvial deposits are more complex than in other parts of the site; sediments range from coarse to very fine, indicating that depositional processes characteristic of both the Wisconsin River and Duck Creek have formed these deposits.

The cooling lake was designed to dissipate the waste heat, which is discharged from the plant after steam has been used to generate the electricity. The 200-ha lake has an effective length of 5,500 m. The lake and the ashpit were formed by dikes 5 m high constructed entirely of local silty sand. The western dikes around the cooling lake were lined with bentonite. Water is pumped into the cooling lake from the Wisconsin River at an average rate of 50,000 m^3/day. Of this water 20% discharges into the ashpit, 40% is lost by ground-water seepage, and 40% is lost by evaporation.

Hot water is discharged at a rate of 1×10^6 m^3 into the north end of the lake east of the central divide (Figure 3). Water circulates in a clockwise direction and is withdrawn from the north end of the lake west of the central divide. Average residence time of a water particle in the lake is 5 days. Water temperaturess at the discharge point average 10-15° C higher than temperatures at the intake and decrease exponentially with distance from the discharge point. The average annual range of lake temperature is 0-45° C. Temperatures in the lake vary only slightly with depth.

Cooling towers have been built to dissipate some of the additional waste heat that is being generated by the second unit. However, the cooling lake is projected to receive 80% of the annual heat load. The discharge of the additional waste heat into the cooling lake is estimated to increase lake temperature by approximately 8° C when the cooling towers are not operating (Wisconsin Public Service Commission 1974).

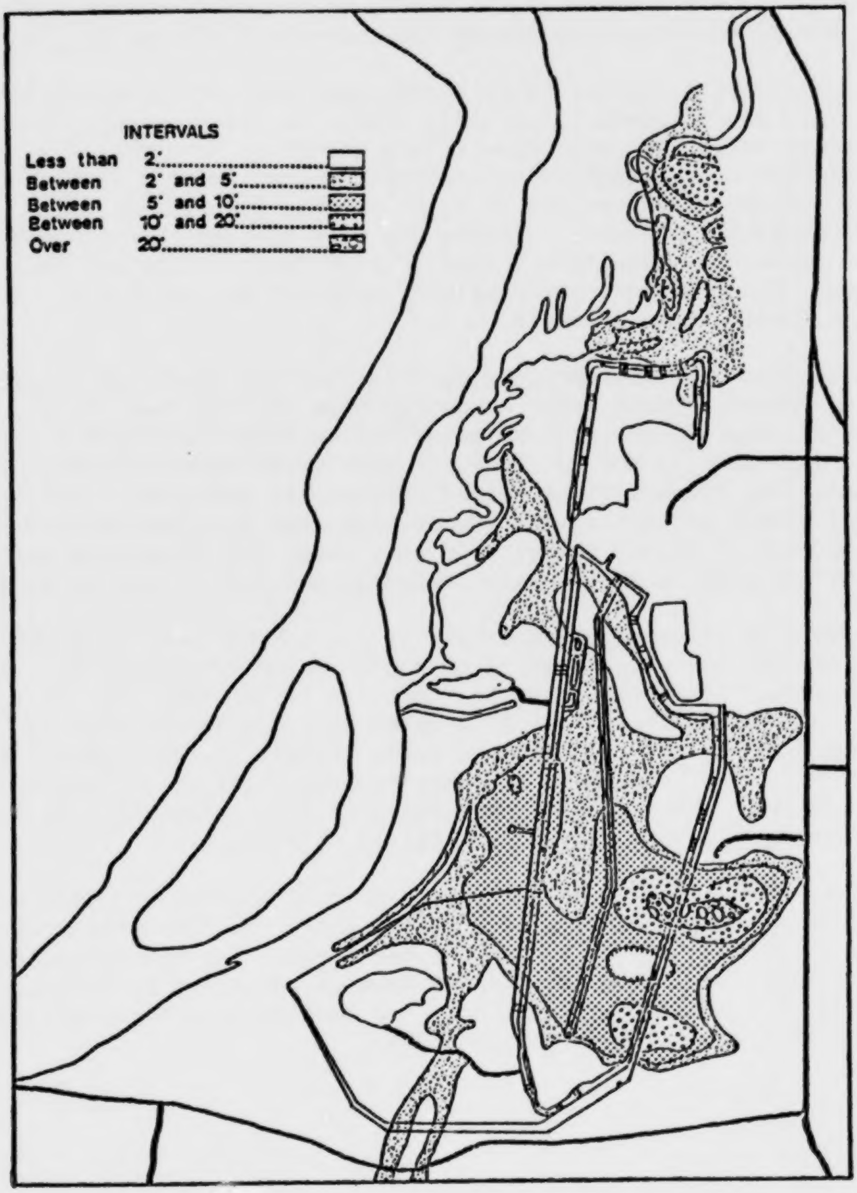
INTERVALS
Less than 2'
Between 2' and 5'
Between 5' and 10'
Between 10' and 20'
Over 20'

Figure 8. Peat thicknesses in the subsurface at the site of the Columbia Generating Station.

SECTION 4

THE IMPACT OF A POWER PLANT ON THE GROUND-WATER SYSTEM OF A WETLAND

CONFIGURATION OF THE GROUND-WATER FLOW SYSTEM

The first goal of the hydrogeologic investigation at the site of the Columbia Generating Station was to determine the impact of the station on ground-water flows, temperature, and chemistry in the adjacent wetland (Anderson and Andrews 1977, Andrews and Anderson 1978). To determine the physical setting in the ground-water system of the wetland before operation of the plant, data were collected beginning in the summer of 1971. The position of the water table was monitored in 80 small-diameter observation wells, and vertical head gradients were measured in 19 nested observation wells.

Two-dimensional steady-state models of several representative vertical cross sections oriented perpendicular to the western dike of the cooling lake were generated. (Figure 6 illustrates two of these cross sections.) These models were then combined to construct a quasi-three-dimensional model of the flow system. This model was used to simulate the head distribution before and after filling of the lake and to compute the total ground-water discharge to the wetland in the area affected by the lake. The vertically oriented cross section models were patterned after the regional ground-water model of Freeze and Witherspoon (1966). The finite difference equations were solved by using the method of successive over-relaxation. A 25-by-50 node grid was used for all simulations. Horizontal nodal spacing ranged from 15 m near the dike to 120 m farther from the dike, and vertical spacing ranged from 0.6 m near the surface to 30 m at depth. Details of the modeling procedure can be found in Andrews (1976).

Field data were used to check the validity of the model, and the model then computed ground-water discharge to the wetland. The simulated head distribution in cross-section A-A' of Figure 6 before filling of the cooling lake is shown in Figure 9. The ground-water flow system was modeled to a depth of 150 m, but for clarity only the upper 30 m are shown. Horizontal flows ranged from 0 to 4.1 cm/day. Vertical flows were mostly toward the surface at rates of 0-0.16 cm/day. Approxmiately 300 m west of what is now the western dike, vertical flows were as high as 0.53 cm/day. Total discharge to the wetland in the cross section shown in Figure 9 was 1.00 m^3/day per meter dike length, of which 0.43 m^3/day per meter discharged in the area of high vertical flows shown in the figure.

A steady-state model was used to simulate pre-lake conditions, although the water table fluctuated about 0.5 m/yr in the lowlands and as much as

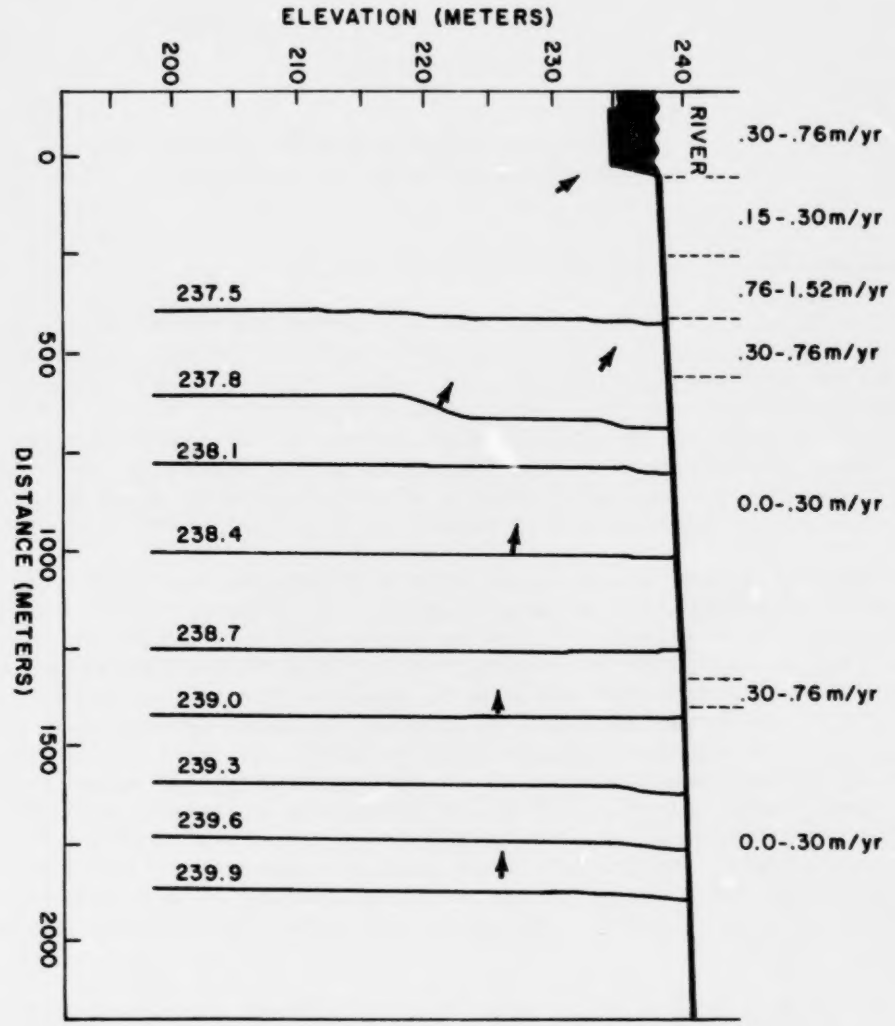

Figure 9. Simulated head distribution in cross-section A-A' of Figure 6 before filling of the Columbia cooling lake. Equipotential lines are labeled in meters. Vertical discharge rates are given across the top of the figure.

2 m/yr in the uplands. However, the water-table gradient remained nearly constant throughout the year, except in the upland areas where the gradient increased markedly during periods of recharge. The head distribution presented in Figure 9 approximates the average annual head distribution.

The initial filling of the cooling lake began 13 June 1974 with water pumped from the Wisconsin River. Pumping stopped on 17 July, when the level was 0.5 m below the fill line. By 1 September the lake was almost empty, and on 4 November 1974 pumping resumed. The water level reached the fill line on 2 January 1975. Water levels in observation wells monitored during the filling of the lake showed increases in vertical gradients in the area west of the cooling lake (Andrews 1976).

The filling of the lake greatly altered the head distribution in the ground-water system (Figure 10). What had been one ground-water system became three systems. Ground water that formerly discharged to the wetland began to discharge near the drainage ditch east of the cooling lake. The lake became the only source of water for the discharge area west of the lake. For the cross section in Figure 10, the simulation predicted that the ground-water discharge to the wetland west of the dike was 3.5 m^3/day per meter and that the flow into the drainage ditch was 3.7 m^3/day per meter. As in the pre-lake simulation, vertical discharge rates were highest approximately 240-300 m west of the dike. In this main discharge area vertical flow rates varied from 1.02 to 1.34 cm/day. Elsewhere, vertical flows ranged from 0.24 to 0.94 cm/day. The position of the main discharge area varied with the cross section modeled. A comparison of total discharge rates to the wetland for Figure 9 and Figure 10 shows that discharge to the wetland in that cross section after filling of the lake was 3 1/2 times greater than discharge before filling.

The effect of the cooling lake on the head distribution in several other cross sections was also simulated. In all simulations the horizontal component of flow was assumed to be perpendicular to the dike. Changes in configuration of the flow system in other cross sections modeled were similar to the changes shown in Figure 9 and Figure 10. In some cross sections, however, the discharge to the wetland was 15 times greater after the filling of the cooling lake. Total discharge to the area of the wetland affected by the lake was estimated to be six times greater after filling.

The assumption of steady-state conditions after filling of the cooling lake is acceptable in view of the stabilization of ground-water levels within a month of the filling of the lake. However, since the water in the cooling lake is warmer than the ground water in the system before the lake was filled, the ground-water system is not yet in equilibrium with the altered thermal regime. The model allows for variation in hydraulic conductivity as a result of differences in ground-water temperatures. For the simulation presented in Figure 10, the temperatures in the ground-water system were asssumed to vary from 26° C directly beneath the cooling lake to 10° C at a depth of 20 m below the lake. The relationship between ground-water temperature and ground-water flows is discussed in section 5.

From the simulations and the supporting field evidence, the following observations can be made. On the east and south sides of the cooling lake,

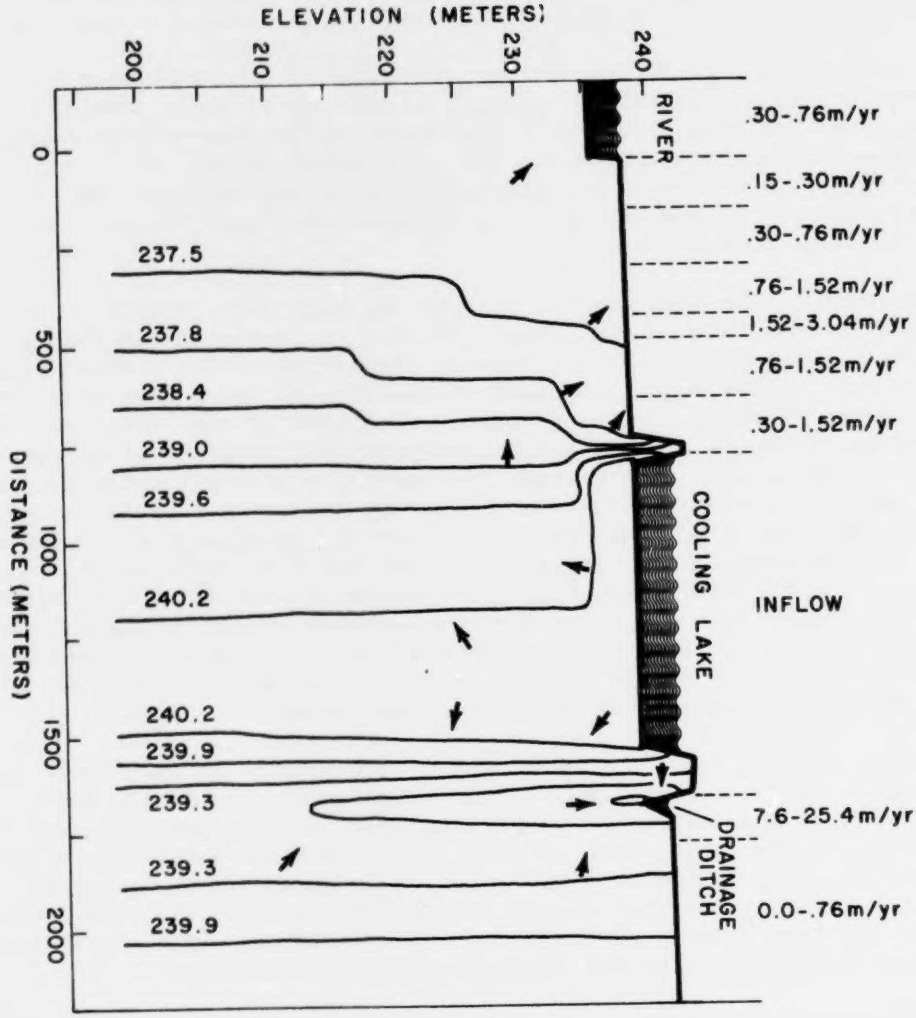

Figure 10. Simulated head distribution in cross section A-A' of Figure 6 after filling of the Columbia cooling lake. Equipotential lines are labeled in meters. Vertical discharge rates are given across the top of the figure.

the drainage ditch effectively limited the impact of the lake on the ground-water system to a relatively narrow zone. On the north and northeast sides of the lake where there is no drainage ditch, the filling of the cooling lake resulted in a rise of ground-water levels of almost 1 m. This rise reversed flow in this area such that water now flows from the lake eastward beneath the coal pile and discharges in the wetland area east of the site. The average annual seepage from the cooling lake is estimated to be 0.18 to 0.27 m^3/s. Flow rates fluctuate about 20% during the year as a result of changing ground-water temperatures.

The average annual seepage from the cooling lake will increase by approximately 20% with both generating units in operation. The discharge rates will also increase because of the formation of springs in the wetland west of the cooling lake. Springs have formed since the filling of the cooling lake, because vertical hydraulic gradients in many areas near the dike exceeded 1 m/m, which approximates the critical gradient for the onset of heaving in unconsolidated materials. In addition, increased temperatures in the marsh have speeded up the process of peat decomposition, and deeper water levels in the marsh have resulted in the floating of parts of the peat mat. The result of both these processes is a decrease in the total load on the confining silt-clay layer, a decrease that enhances the onset of heaving. Heaving is not predictable, and therefore accurate estimates of its rapidity and of its impact on flow rates are impossible. Potentiometric data from observation wells in the marsh indicate that the springs that have formed during the first 2 yr since the cooling lake was filled have not significantly altered flow rates. Increases in flow rates caused by these processes will probably not exceed 8% during the next 10 yr.

WETLAND WATER LEVELS

The wetland west of the cooling lake lies in two distinct drainage basins, each of which is connected by distinct channels to tributaries of the Wisconsin River. The wetland area adjacent to the cooling lake and north of a line 350 m south of the intake channel drains to Duck Creek, and the area south of this line drains to Rocky Run Creek via a small channel (Figure 3). Ground-water inflow to the northern basin averages approximately 0.09 ± 0.03 m^3/s, and inflow to the southern basin averages approximately 0.10 ± 0.03 m^3/s.

The water level in the wetland is a function of the ground-water inflow rate, the rate of water leaving the wetland via surface outflow, the rate of precipitation and evapotranspiration, and the wetland basin shape. The rate at which water leaves the wetland is a function of the water level and the shape of the channels draining the wetlands. On a daily basis evapotranspiration from each wetland basin seldom exceeds 0.04 m^3/s. Because of the shape of the channels that drain the basins, a change in outflow from 0.04 to 0.20 m^3/s raises water levels less than 5 cm.

Water levels in the wetland have been almost constant since the cooling lake was filled. The level averages about 10 cm higher than previous levels (Figure 11). Wetland water levels do fluctuate during the winter when ice clogs the drainage channels and during floods on the Wisconsin River.

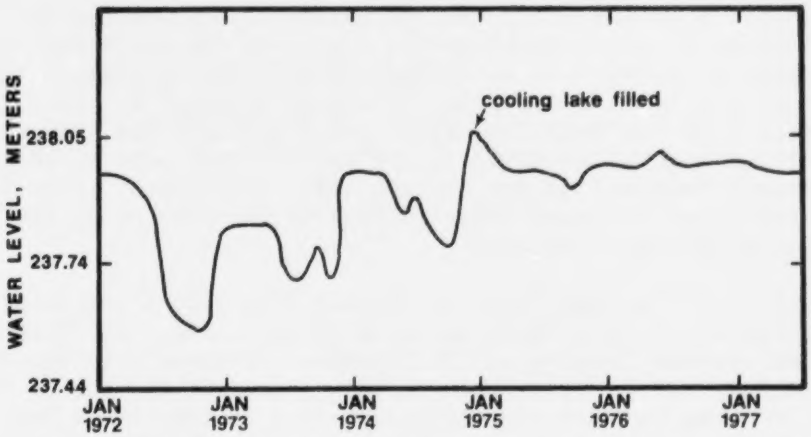

Figure 11. Water levels in the wetland west of the Columbia cooling lake before and after filling of the lake.

Before the cooling lake was filled, ground-water inflow rates in the summer to each of the wetland basins were only 0.2 to 0.3 m^3/s. Evapotranspiration often exceeded inflow, and there was no outflow from the wetlands via the channels. During the late summer water levels were lowered as much as 0.6 m.

The peat substrate in the wetland has been altered by the increased flow rates within the wetland, the increased water temperatures, and the increased water levels. Some peat has been eroded, and new channels have been cut in the wetland as the internal drainage system adjusts to the increased flow. Other parts of the peat mat have floated to the surface and been broken down by various physical and biological processes. Peat decomposition rates have increased because of the increased water temperatures and increased oxygen content of discharging ground water. The plant community is rapidly responding to the new environment as evidenced by the replacement of formerly dominant sedges, Carex lacustris and Carex stricta, by plants more tolerant of deeper water, primarily Sagittaria latifolia and Rumex orbiculata (Willard et al. 1976).

GROUND-WATER TEMPERATURE

Before the cooling lake was filled, the temperature of ground water discharging to the wetland was approxmiately equal to the average annual air temperature (10° C). The cooling lake, now the source of water discharging to the wetland, has an average annual temperature between 10° C and 17° C, depending on location in the lake.

In areas with a layer of relatively high permeability near the dike, discharge rates are high and seepage from the lake has a travel time of less than 1 yr before discharging to the wetland. Consequently, the range of ground-water temperatures is similar to that of the lake water, although the extremes are somewhat attenuated. The temperature of water in the western

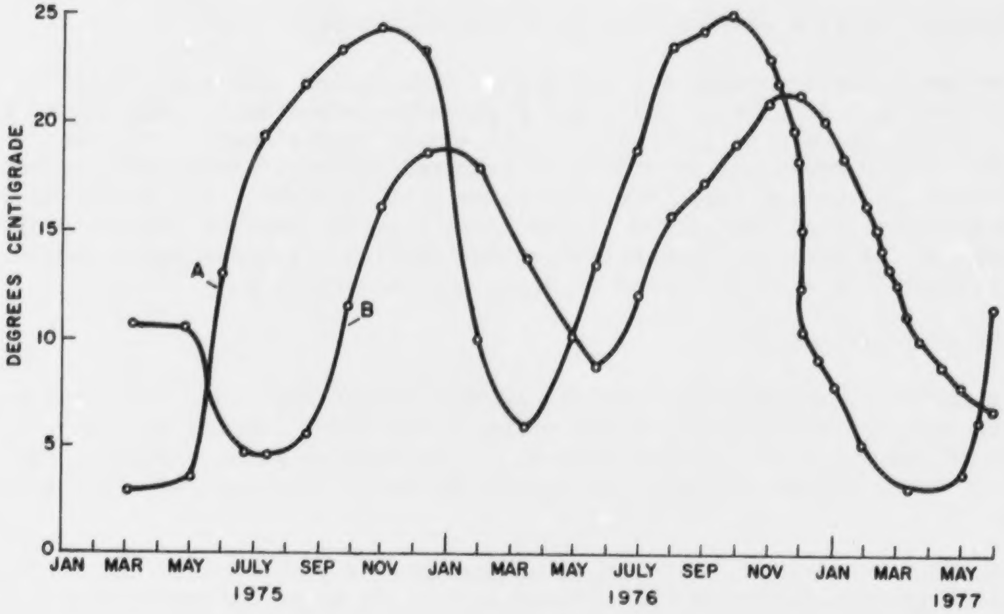

Figure 12. Ground-water temperature variations in wells west of the Columbia cooling lake. Curve A: data from a well cased to 4 m below the surface and located 3 m west of the dike. Curve B: data from a well cased to 2.5 m below the surface and located 60 m west of the dike.

portion of the lake from May 1975 to May 1976 ranged from 1° C to over 30° C. Ground-water temperature in a well 3 m west of the dike varied from 2° C to 25° C (Figure 12, curve A), and in areas west of this well, attenuation of extremes was even greater (Figure 12, curve B).

To measure the heat discharge into the ground-water system, 47 thermistors were installed in the summer of 1976 to depths of 10 m below the surface along cross-section A-A' of Figure 6. Temperature data were collected twice a week to document seasonal trends in the variation of ground-water temperature. The temperature changes observed in the ground-water system are described in detail in section 5. Preliminary calculations based on these thermistors suggest that approximately 16 MW, less than 5% of the heat released to the cooling lake, is discharged to the ground-water system. Ground-water temperatures increased as much as 20° C, however, in some areas west of the cooling lake.

Normally, the species of sedges present in the marsh emerge early and die back late in the growing season. During the winter of 1975, however, sedges in some areas of the marsh turned green in December because of the warm water. In the spring these sedges were dead, possibly because they had used up their stored food reserves. In 1976 these vegetation changes occurred over a considerably larger area (Willard et al. 1977).

RELATIONSHIP BETWEEN GROUND-WATER TEMPERATURES AND FLOW RATES

The ground-water system was not yet in equilibrium with the altered thermal regime. Because ground-water flow rates depend on temperature of the ground water, leakage is greatest in late summer when the water is the warmest. Furthermore, as ground-water temperatures rise in response to heat input from the cooling lake, the flow rates will increase. The ground-water flow model predicted that flows in the cross section shown in Figure 10 will increase by 24% when the system reaches equilibrium. A model which couples ground-water flow with heat flow is discussed in section 5.

CHANGES IN WATER CHEMISTRY

During the preoperational period (August 1972 to September 1974) 144 water samples were taken from observation wells on the site. During the first 7 months of operation 134 samples were collected from 57 wells. Some of the types of water on the site are illustrated by Stiff diagrams in Figure 13 and Figure 14.

Significant changes in the ionic composition of ground water and surface waters in the wetland since the generating station began operation were observed only near the ashpit. Because of the similarity in the chemical composition of cooling-lake water and ground water, no significant changes are likely to be observed in ground-water quality near the cooling lake. If changes do occur, they are likely to be in sodium concentrations caused by changes in ion-exchange processes associated with higher flow rates.

Major changes are likely to occur in ground-water quality near the ashpit because ground water and water in the ashpit differ in quality. Prediction of these changes is not yet possible because water levels in the ashpit have fluctuated widely during the first 2 yr of operation and because the chemical composition of ashpit waters has varied widely. Ground-water flow rates from the ashpit have consistently ranged from 0.3 to 0.6 m^3/s. The filling of the pit with ash has reduced seepage through the bottom, but this reduction has been offset by increased flow through the dikes caused by elevated water levels. Most ground-water flow from the ashpit now occurs through the dikes, and most of the seepage water discharges near the dikes.

Some change has been observed in ground-water quality near the ashpit. Calcium and sulfate concentrations have increased significantly in two wells close to the ashpit dike (Table 2). Changes in water quality have been noted in other wells farther from the dike, but these changes have not been as great (Table 2). The plume of contaminated ground water appears to be confined to a relatively small area near the dikes. However, pronounced increases in specific conductivity in surface waters in the marsh have been measured up to 50 m from the dike, which suggests that water is leaking from the ashpit through the ground-water system and is discharging to the wetland where the water then spreads outward. Head gradients in observation wells support this hypothesis.

The most significant degradation of ground-water quality has occurred in the vicinity of the 17-ha coal pile. Although background sulfate

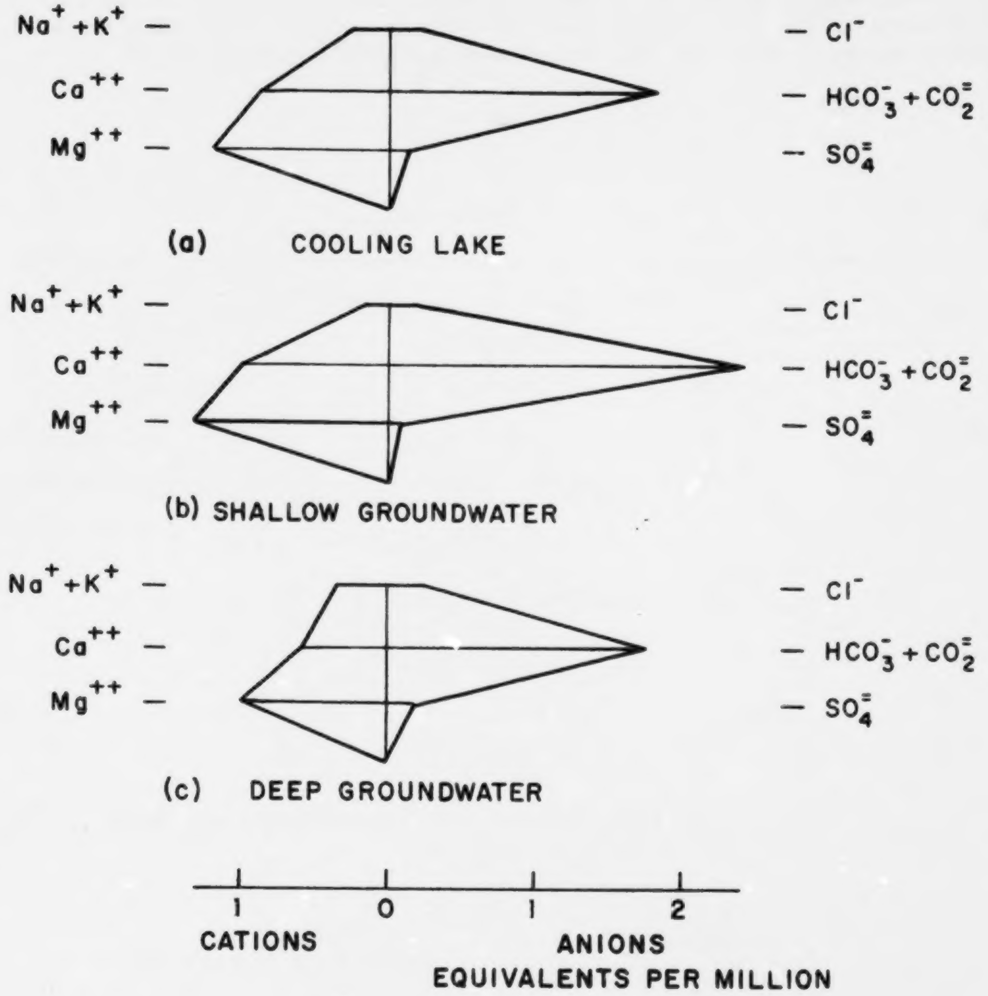

Figure 13. Stiff diagrams of water in the cooling lake (a) and ground water (b and c). Data were collected in fall 1975. Ground water (b) is from a well cased to 2 m below the surface and located 50 m west of the north end of the cooling lake; ground water (c) is from a well cased to 6 m below the surface and located 60 m west of the south end of the cooling lake.

concentrations are less than 20 mg/liter, a plume of contaminated ground water has been created with a sulfate concentration of greater than 1,000 mg/liter. This concentration exists to a depth of more than 30 m near the coal pile and extends more than 100 m to the east of the pile. The plume is gradually spreading east, toward the wetland discharge area (Andrews 1976).

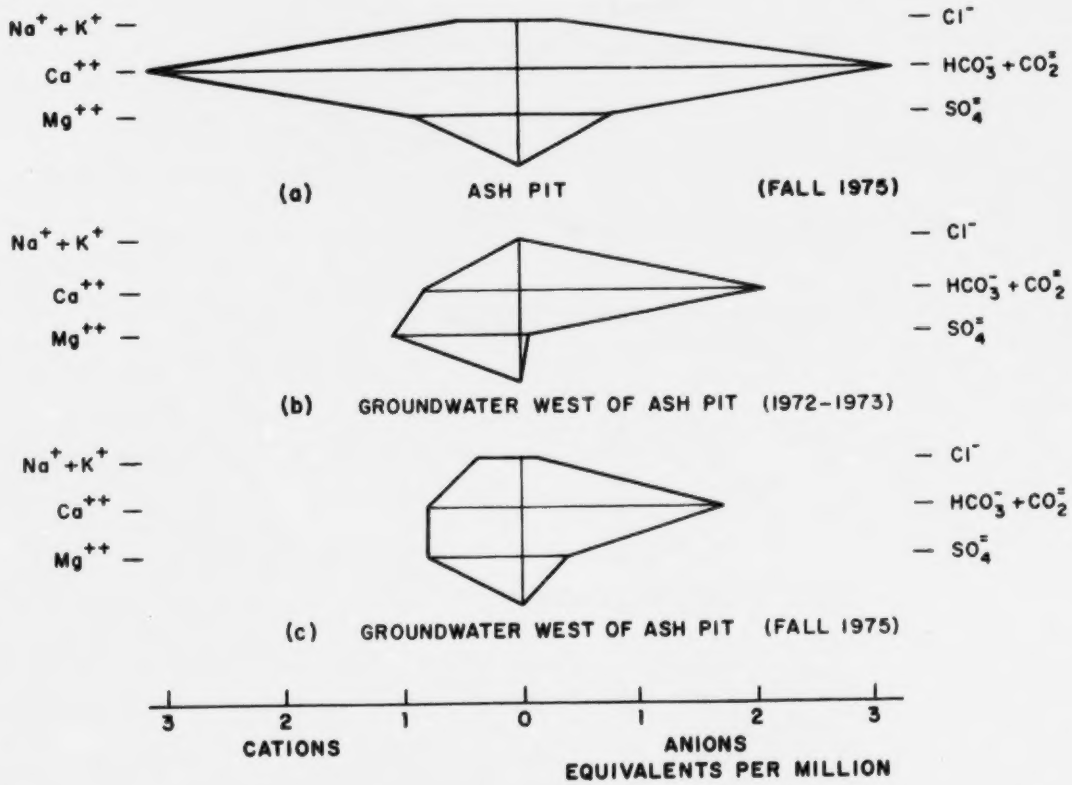

Figure 14. Stiff diagrams of water in the ashpit (a) and in a well cased to 1.5 m below the surface and located 2 m west of the ashpit (b and c). Comparison of (b) and (c) shows the change in water quality after the partial filling of the ashpit.

TABLE 2. SELECTED CHEMICAL CONCENTRATIONS IN GROUND WATER NEAR THE ASHPIT OF THE COLUMBIA GENERATING STATION

A. West of the dike

		1 m west			75 m west		
(m/l)	Ashpit water	1972-73	1975-76	1977	1972-73	1975-76	1977
K^+	2.3	1.2	9.2	6.9	1.2	0.75	0.5
Na^+	10.2	2.2	8.5	13.2	3.6	3.5	4.0
Ca^{++}	77.8	13.9	16.9	21.6	31.6	28.8	24.3
Mg^{++}	10.6	11.1	9.8	14.7	20.3	22.0	16.9
SO_4	44.6	5.4	18.5	10.5	0.8	0.2	0.1
Cl^-	10.6	3.4	4.5	6.5	4.49	2.64	5.0

B. North of the dike

		1 m north			25 m north		
(m/l)	Ashpit water	1972-73	1975-76	1977	1972-73	1975-76	1977
K^+	2.4	1.7	0.75	1.5	2.0	0.75	0.5
Na^+	10.3	4.3	2.6	5.1	5.2	3.1	3.7
Ca^{++}	54.7	18.8	23.3	37.0	22.0	22.6	21.5
Mg^{++}	4.3	17.3	22.5	33.8	18.0	18.8	25.8
SO_4	36.0	7.7	9.6	22.0	9.7	9.3	14.0
Cl^-	10.5	2.8	4.2	8.5	7.2	3.2	3.5

SECTION 5

THERMAL ALTERATION OF GROUND WATER CAUSED BY SEEPAGE FROM THE COOLING LAKE

In addition to monitoring and modeling ground-water flow rates near the Columbia Generating Station, we investigated the effect of the station's cooling lake on ground-water temperatures (Andrews and Anderson 1979). Ground-water temperatures in the vicinity of the cooling lake were monitored in detail in the field for 1.5 yr. The presence of the cooling lake, which loses water to the ground-water system at a rate of 2×10^4 m^3 per day, has created a zone of thermally altered ground water, but the zone is confined to a relatively small area hydraulically downgradient from the cooling lake.

A mathematical model was developed to simulate the response of subsurface temperatures to seasonal changes in lake and air temperatures. To create the model, equations describing ground-water flow were coupled with equations for heat flow in the subsurface. An equation describing the rate of heat loss from the marsh surface was used as one of the boundary conditions for the heat-flow model. The model was solved numerically by using the finite element technique. The model assumed that the flow of heat and water from the cooling lake occurred in planes normal to the plane of the dike. The flux of water and heat was modeled in seven of these planes (Figure 15), which represent two-dimensional vertical cross sections of the ground-water system. Results from each of the cross sections were combined to obtain the total flow of heat and water outward from the west dike of the cooling lake.

Simulated temperature patterns agreed well with field data, but were very sensitive to the distribution of subsurface lithologies. Results from a predictive simulation suggest that operation of the second 500-MW unit will increase ground-water temperatures less than 5° C at distances greater than 15 m from the cooling lake. The results of this study suggest that the potential for significant thermal alteration of surface water bodies located in ground-water discharge areas is slight.

MATHEMATICAL MODEL

Governing Equations

The equation describing the two-dimensional flow of water through a nonhomogeneous aquifer may be written

$$\frac{\partial}{\partial x_i}\left(K_{ij}\frac{\partial \phi}{\partial x_j}\right) = 0, \ i, \ j = 1, 2 \quad , \qquad (1)$$

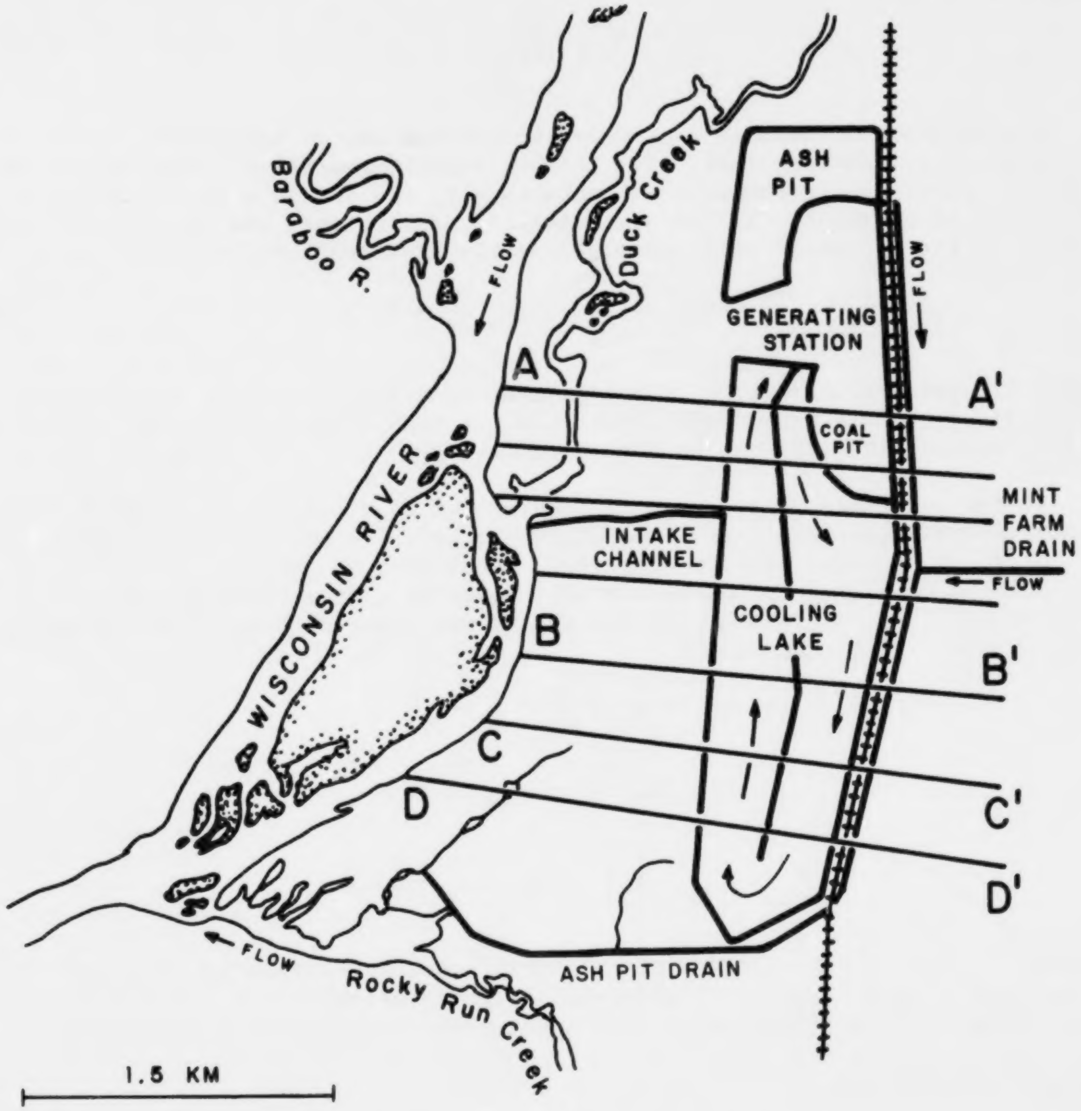
Baraboo R.
FLOW
Duck Creek
ASH PIT
FLOW
GENERATING STATION
A
A'
COAL PIT
WISCONSIN RIVER
MINT FARM DRAIN
INTAKE CHANNEL
FLOW
COOLING LAKE
B
B'
C
C'
D
D'
ASH PIT DRAIN
FLOW
Rocky Run Creek
1.5 KM

Figure 15. Location of seven cross sections of the Columbia site for which ground-water temperature distributions were simulated.

where K_{ij} = hydraulic conductivity, L/t; ϕ = head, L; and x_1, x_2 = cartesian coordinates, L. The ground-water velocity or specific discharge can then be determined from

$$q_i = -K_{ij} \frac{\partial \phi}{\partial x_j} \quad . \tag{2}$$

The movement of heat in a ground-water system can be described mathematically assuming that: (1) Thermal equilibrium between the liquid and the soil particles is achieved instantaneously, (2) the densitiy of the soil particles is constant, (3) the heat capacity is constant, and (4) the chemical system is inert. Under these assumptions the heat transport equation is

$$\frac{\partial}{\partial x_i}\left(D_{ij}\frac{\partial T}{\partial x_j}\right) - \rho C_w \frac{\partial (q_i T)}{\partial x_i} - \rho C_s \frac{\partial T}{\partial t} = 0, \; i, \; j = 1, \; 2 \quad , \tag{3}$$

where T = temperature, T; D_{ij} = coefficient of dispersion, H/TtL; ρC_w = heat capacity of the saturated media, H/L^3T; ρC_w = heat capacity of water, H/L^3T; and q_i = specific discharge in direction x_i, L/t.

If the medium is isotropic, the coefficient of dispersion is a second-rank tensor (Scheidegger 1961) composed of two parts: (1) The thermal conductivity of the medium and (2) the coefficient of mechanical dispersion, which represents the mixing caused by the heterogeneity of the velocity field. In this analysis the coefficient of dispersion was assumed to have the following form (Reddel and Sunada 1970):

$$\begin{aligned} D_{22} &= K^t_{22} + \alpha_L \frac{q_2 q_2}{q} + \alpha_T \frac{q_1 q_1}{q} \quad , \\ D_{11} &= K^t_{11} + \alpha_L \frac{q_1 q_1}{q} + \alpha_T \frac{q_2 q_2}{q} \quad , \\ D_{21} &= D_{12} = K^t_{12} + (\alpha_L - \alpha_T) \frac{q_1 q_2}{q} \quad ; \end{aligned} \tag{4}$$

where $q = (q_1^2 + q_2^2)^{1/2}$; $K_{11} = K_{22}$ = thermal conductivity, H/TtL (thermal conductivity is assumed to be independent of direction); αL = longitudinal (horizontal) dispersivity, L; and αT = transverse (vertical) dispersivity, L.

Boundary Conditions

For the ground-water flow model, heads were specified at the water table and along the vertical boundary east of the cooling lake. The model can be modified to consider the time-dependent case of a moving water table, but for the problem considered here fluctuations of the water table were negligible. No-flow boundaries were specified along the lower boundary and at the Wisconsin River.

For the heat-flow model, a heat flux was specified along the upper boundary and along the vertical boundary east of the cooling lake. The fluxes of heat across these boundaries were specified to be proportional to the water flow across the boundaries. Along that part of the upper boundary representing the marsh, a heat flux resulting from atmospheric exchanges was specified. The other two boundaries were specified as no-flow boundaries.

The process of heat exchange between the marsh surface and the atmosphere is complex, and mathematical representations are necessarily rough approximations. For this study the boundary between the marsh and the atmosphere was assumed to occur in the vegetation at the height where transfer of heat and water vapor by turbulent exchange becomes effective. This level, known as the zero-displacement plane, was assumed equal to 0.63 of the vegetation height. Heat flow from the ground surface to the zero-displacement plane was assumed to occur by conduction and free convection. The heat flow at this boundary can be expressed as

$$q_s = h_\gamma \, (T_m - T_a) + h_m \, (e_m - e_a)/\gamma + (1-a)\varepsilon\sigma(T_m^4 - T_{sky}^4) - (1-a)\alpha_s q_{sun} \ , \tag{5}$$

where T_{sky} = effective sky temperature, T; T_m = temperature at the marsh surface, T; T_a = air temperature above the plane of zero displacement, T; q_s = heat flux from the marsh boundary, H/tL^2; h_m = mass transfer coefficient, H/L^2Tt; $h\gamma$ = heat transfer coefficient, H/L^2Tt; γ = psychrometer constant, assumed equal to 0.66 mbar/T, M/t^2LT; e_m = saturation vapor pressure at the marsh boundary temperature, M/t^2L; e_a = partial vapor pressure of water in air, M/t^2L; ξ = marsh boundary long-wave emissivity, dimensionless; σ = Stefan-Blotzmann constant, H/T^4tL^2; α_S = marsh boundary solar absorptivity dimensionless; q_{sun} = solar flux incident on the marsh boundary, H/tL^2; and a = percentage of open water in the marsh, dimensionless.

In addition to areas of dense vegetation, the marsh also contains some areas of open water. For the latter it was assumed that radiative exchange and solar absorption occur at the surface.

The heat and mass transfer coefficients (h_v and h_m) are equal if the roughness height is the same for both. According to Mitchell et al. (1975) these coefficients are given by an equation of the form

$$h_m = h_v = k^2 \ \rho C_p u/\ln(Z_a - d/Z_o + 1) \quad , \tag{6}$$

where k = Karman coefficient, dimensionless; ρC_p = heat capacity of air, H/L^3T; u = air velocity, L/t; Z_a = reference height, L; Z_o = surface roughness height, L; and d = height of the zero-displacement plane, L. The surface roughness height can be estimated from vegetation height by using the relationship given by Tanner and Pelton (1960), $\log Z_o = 0.997 - \log 0.833$, where l is the average height of the vegetation.

Following the method of Jobson (1972), we used daily average values for air temperature (T_a) and vapor pressure (e_a) in Eq. (6). Moreover, Eq. (6) can be simplified when temperature data at the marsh surface are available, and the terms involving higher order dependence on T_m are approximated by linear relations. Then, the heat transfer from the marsh surface is proportional to the temperature difference between the marsh surface and the air temperature at 2 m above the surface. The constants of proportionality were determined for a cross section for which data had been collected, and these values were used when simulating heat flow in the other cross sections. The heat-flow rates calculated by this method agreed well with values calculated by using Eq. (6).

Solution Procedure

The finite element method (Zienkiewicz 1977, Pinder and Gray 1977) was used to solve Eq. (1) and Eq. (3) subject to the boundary conditions described in the previous section. Each cross section modeled was divided into 100-150 quadrilateral elements. The procedure used to link the equations was: (a) Eq. (1) was solved for head at each node and Eq. (2) was solved for velocity; (b) Eq. (3) was solved for temperature at each node; (c) the solution to Eq. (3) was stepped forward in time by the Crank-Nicolson approximation for the time derivative for a specified number of time steps; (d) the hydraulic conductivity distribution, which is a function of temperature, was adjusted for the new temperature distribution; and (e) steps (a)-(d) were repeated.

Required Parameters

The physical properties required as input parameters are thermal conductivity, specific heat capacity, hydraulic conductivity, and porosity for each material type in the subsurface and dispersivity.

The values of the parameters for each of the subsurface materials used in the simulations are listed in Table 3. Thermal conductivities were determined by the needle probe technique (Von Herzen and Maxwell 1959). Heat capacities were determined by standard additive techniques (Van Wijk and deVries 1963). Horizontal hydraulic conductivities were estimated from aquifer test data, slug tests, and grain-size analyses. Vertical hydraulic conductivity in each element was assumed to be one-tenth of the horizontal hydraulic conductivity. Porosities, which were needed to estimate pC_s, were approximated from grain-size analyses.

Longitudinal dispersivity was assumed constant for all elements and was estimated by a trial-and-error adjustment procedure. Lateral dispersivity is usually assumed to be one-quarter to one-tenth of longitudinal dispersivity (Cherry et al. 1975). For the simulations reported here, the ratio of longitudinal to transverse dispersivity was assumed to be 4. The field data were best simulated by using a longitudinal dispersivity of 10 cm. When higher values were used the amplitude of the annual temperature wave observed in the field could not be reproduced. In Figure 16 the observed temperatures 2 m west of the dike at a depth of 3 m are plotted along with the simulated temperatures obtained by using various values for longitudinal dispersivity.

Values for longitudinal dispersivity reported in the literature range from 0.1 cm for laboratory studies with homogeneous sands (Hoopes and Harleman 1967) to over 100 m for field studies (Bredehoeft et al. 1976). The value of dispersivity is known to be related to media inhomogeneities and the scale of the problem, but the exact relationship is unknown. For the problem described

TABLE 3. PARAMETERS USED IN THE SIMULATIONS FOR THE MODEL DESCRIBING THERMAL ALTERATION OF GROUND-WATER

Lithology	Horizontal hydraulic conductivity (m/day)	Thermal conductivity (10^{-1} cal/m sec°C)	Heat capacity (cal/cm^3°C)	Porosity
Medium coarse sand with gravel	30	0.45	0.64	0.33
Fine to very fine sand	10	0.51	0.67	0.39
Weathered sandstone	8	0.51	0.67	0.39
Sandstone	3.5	0.64	0.60	0.26
Peat	3.0	0.12	0.91	0.75
Sandy silt to sandy clayey silt	0.4	0.42	0.72	0.48
Gray sandy silt with organic matter	0.04	0.42	0.72	0.48
Varbed clay with sand seams	0.03	0.40	0.74	0.52

RESULTS

Field Data

Temperatures were monitored twice a week at 48 points in cross-section A-A' in Figure 15 for 13 months and at 18 points in cross-section B-B' in Figure 15 for 6 months. Temperatures were recorded at the surface and at depths to 10 m below the surface. In situ thermistors, hardwired to a central control box, were used to obtain ground-water temperatures in cross-section A-A'. Temperatures in cross-section B-B' were measured by lowering a thermistor probe into three 3.175-cm wells. The thermistors were calibrated to ±0.1° C. A high-resolution digital ohmmeter was used to measure the resistance of the thermistors. Surface-water temperatures at two sites along cross-section A-A' were monitored continuously with liquid expansion thermographs.

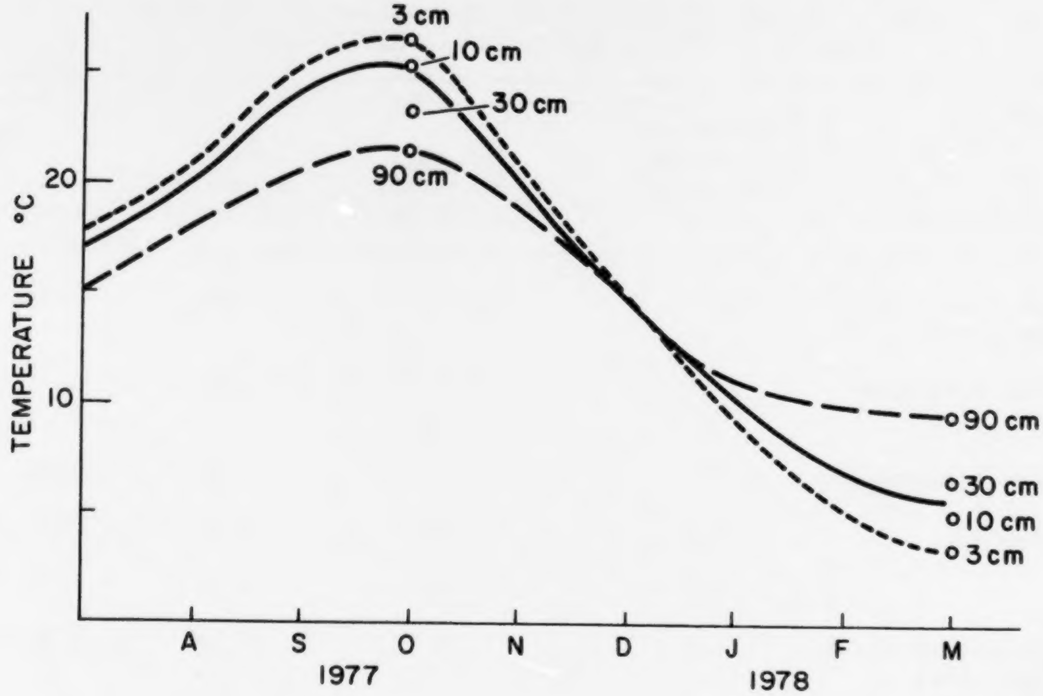

Figure 16. Observed temperatures in the subsurface west of the Columbia cooling lake and temperatures simulated by the mathematical model. The ratio of longitudinal dispersivity (α_L) to transverse dispersivity (α_T) is set at 4. (——) is observed temperatures with α_L = 3 cm. (— —) is simulated temperatures with α_L = 90 cm. (o) indicate maximum and minimum simulated temperatures for α_L/α_T = 4 with L equal to 3, 10, 30, and 90 cm.

The temperature distributions recorded in cross-sections A-A' and B-B' at 3-month intervals are shown in Figure 17 and Figure 18. Dissimilarities in the temperature distributions recorded in the two cross sections are attributed to differences in the subsurface materials, which result in different distributions of ground-water velocity. Average ground-water velocity is slower in cross-section B-B' by a factor of 2. The fluctuations in average lake temperature and the seasonal fluctuations of ground-water temperature at several distances from the dike along cross-section A-A' at a depth of 4.5 m are illustrated in Figure 19. The lag time between the occurrence of the maximum temperature in the ground water and the maximum cooling-lake temperature gradually increases, and the amplitude of the fluctuation decreases with distance from the dike.

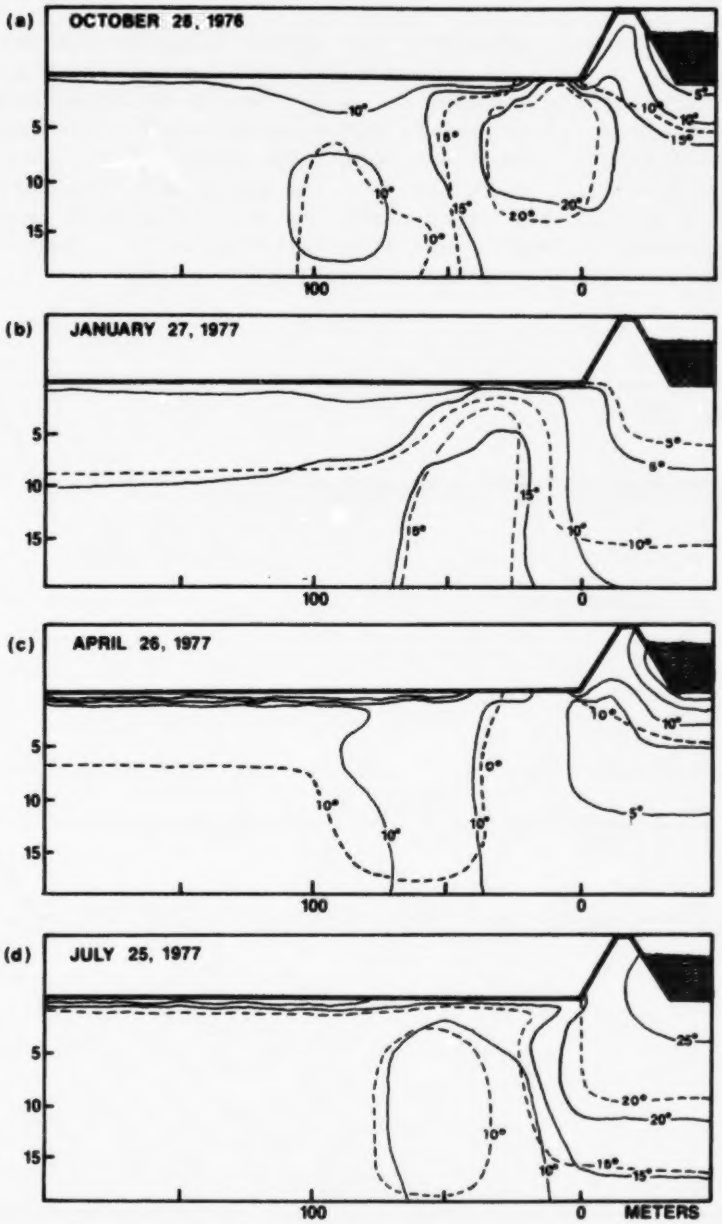

Figure 17. Observed (———) and simulated (----) ground-water temperature distributions in cross-section A-A' of Figure 15.

Simulations

Good agreement between simulated and actual temperatures was obtained when the model was used to simulate temperatures in cross-sections A-A' and B-B' (Figure 17, Figure 18, Figure 20), but only because detailed data on the subsurface distribution of materials had been obtained. The temperature patterns observed in the subsurface were very sensitive to the distribution of layers with low hydraulic conductivity. The model was most sensitive to the extent and depth of the clay layer under the peat (Figure 8) and to a clay layer that is present in some areas at a depth of 6-8 m. Over 70 borings were made in the marsh to determine the subsurface lithology.

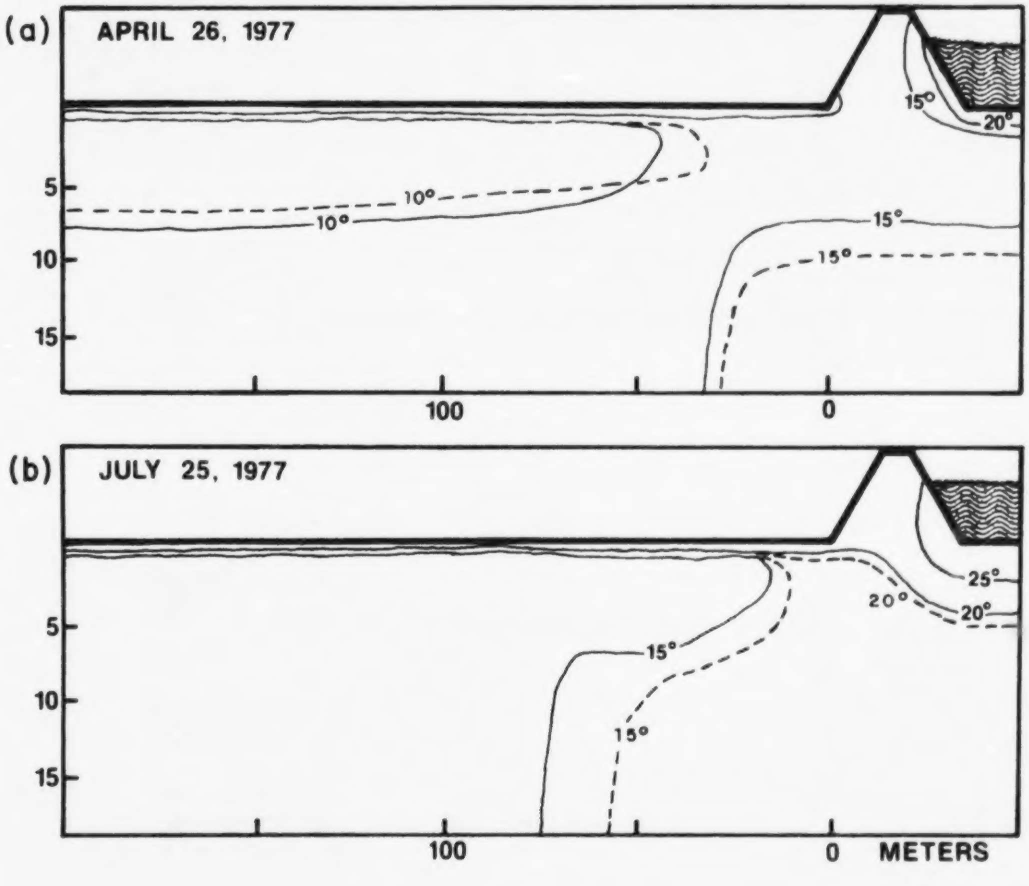

Figure 18. Observed (——) and simulated (----) ground-water temperature distributions in cross-section B-B' of Figure 15.

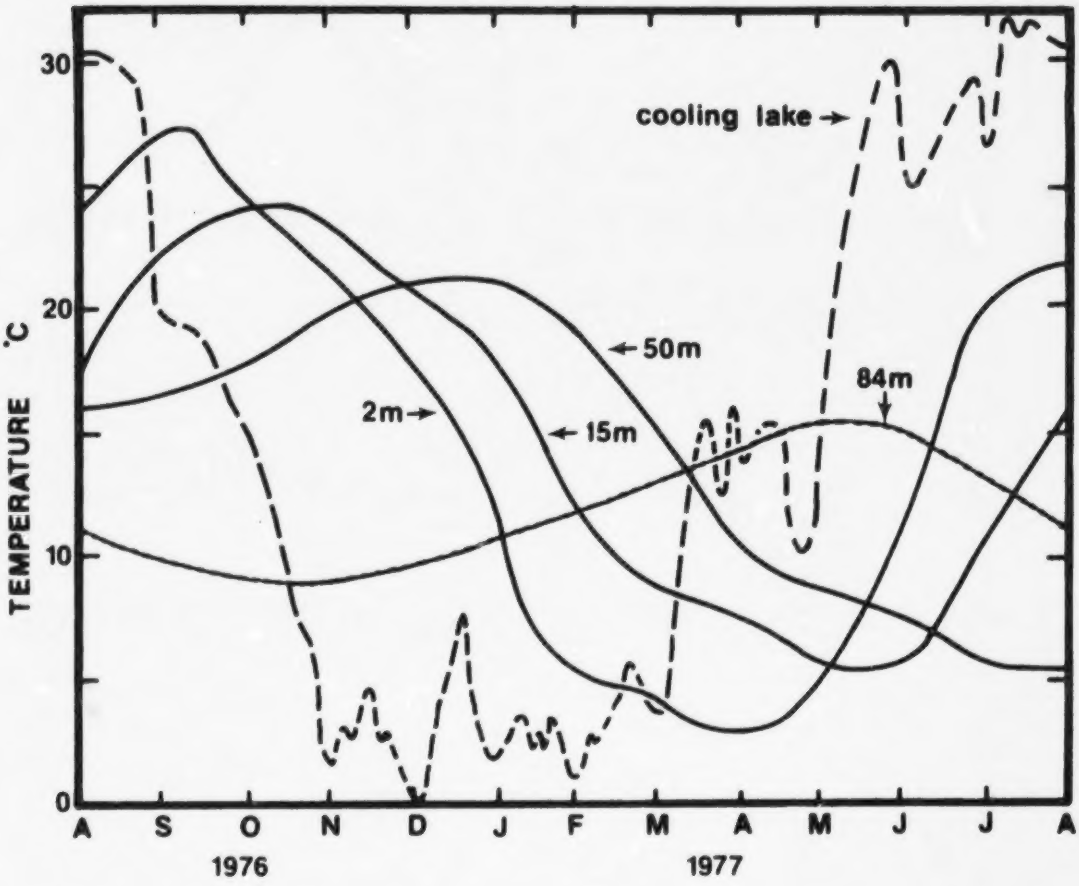

Figure 19. Seasonal fluctuations of ground-water temperature in cross-section A-A' of Figure 15 at a depth of 4.5 m at distances of 2, 15, 50, and 84 m west of the cooling lake dike. The fluctuations of average lake temperature are also shown (----).

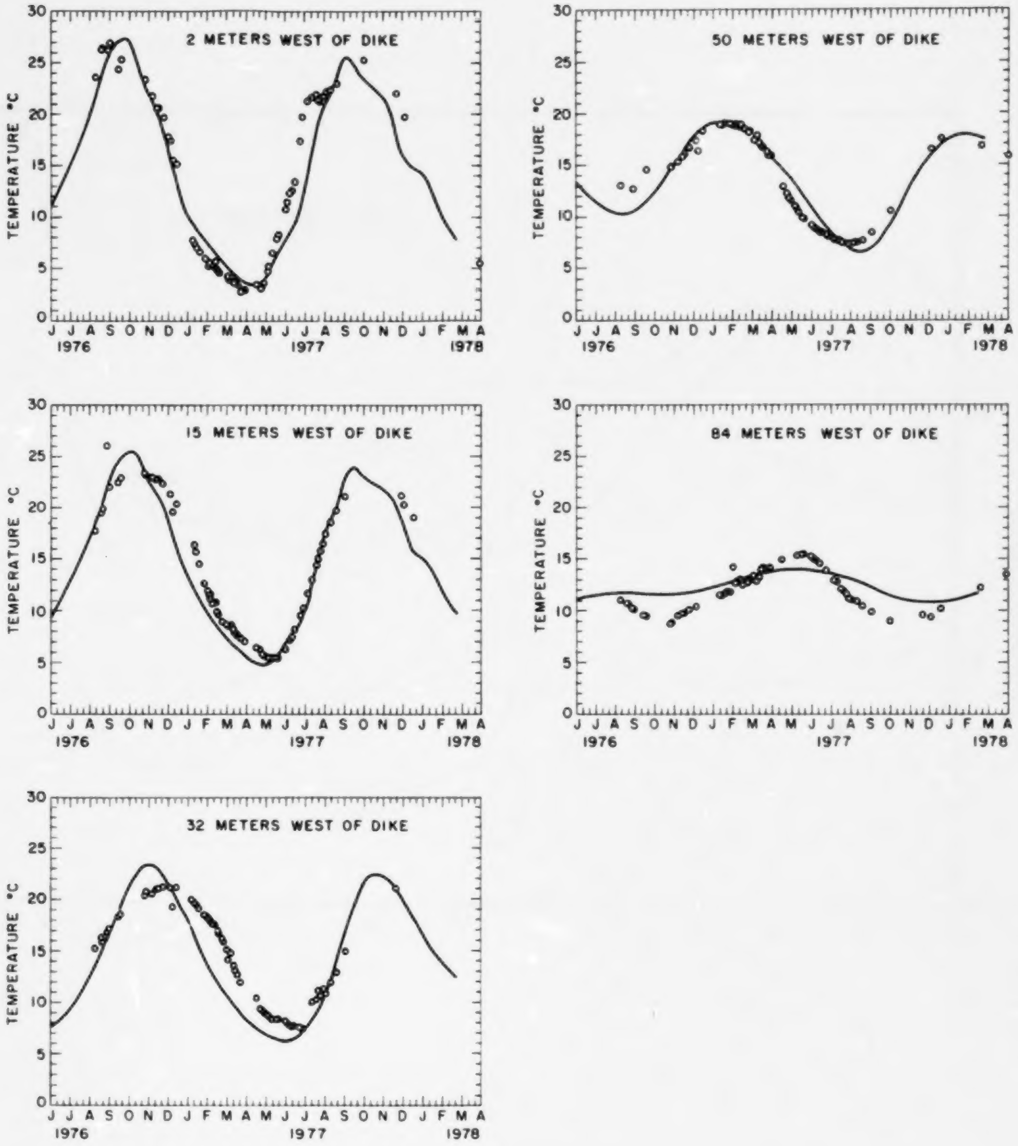

Figure 20. Simulated temperatures (———) and observed temperatures (o) in cross-section A-A' of Figure 15 at a depth of 3.28 m at 2, 15, 32, 50, and 84 m west of the cooling lake dike.

SECTION 6

LONG-TERM TEMPERATURE CHANGES IN THE GROUND WATER OF THE WETLAND

SIMULATION STUDIES

Simulations were used to predict the long-term changes in substrate temperature that may occur in the wetland adjacent to the Columbia Generating Station. Wetland water levels and ground-water flow rates into the wetland had stabilized during the first 2 yr of operation of the generating station, but neither ground-water temperatures nor wetland vegetation (Bedford 1977) had yet reached equilibrium. Prediction of the nature and magnitude of change in vegetation was not possible without an understanding of the probable changes in substrate temperatures.

The model that was described in section 5 to simulate the response of subsurface temperatures to changes in the temperatures of the cooling lake and the air was used to simulate seasonal temperature patterns for the 12-yr period from November 1974 to January 1987. The governing equations, boundary conditions, microclimatic model, and parameters remained the same in the long-term simulation. Changes in temperature for the 12 yr were modeled in the same two planes, which represent two-dimensional vertical cross sections of the ground-water system (A-A' and B-B' in Figure 15).

Cooling-lake temperatures and air temperatures for the 10-yr period 1978-87 were synthesized by repeating five times the actual temperatures recorded from January 1976 to January 1978. For the period after April 1978 when two generating units were assumed to be operating, the temperatures were adjusted to account for the added heat load. When the temperature at the generating station outlet rose above 40° C, it was assumed that the entire heat load from the second generating unit was dissipated in the cooling towers. The cooling-lake temperatures at the generating station inlet for the period 1975-87 are shown in Figure 21.

The simulations predict that temperatures at a depth of 0.6 m will not fall below 8° C within 200 m of the dike by 1987 and that peak temperatures near the dike will be 10-15° C above normal and will occur in October and November rather than in August. The subsurface stratigraphy of the site is such that major changes in near-surface temperatures will only occur within 350 m of the dikes, but if the stratigraphy were different the effects could extend to much greater distances from the cooling-lake dikes.

The temperature changes simulated in the ground-water system for the period 1975-87 are only an approximation of the actual temperatures that may exist in the ground-water system during this period. Lake and air

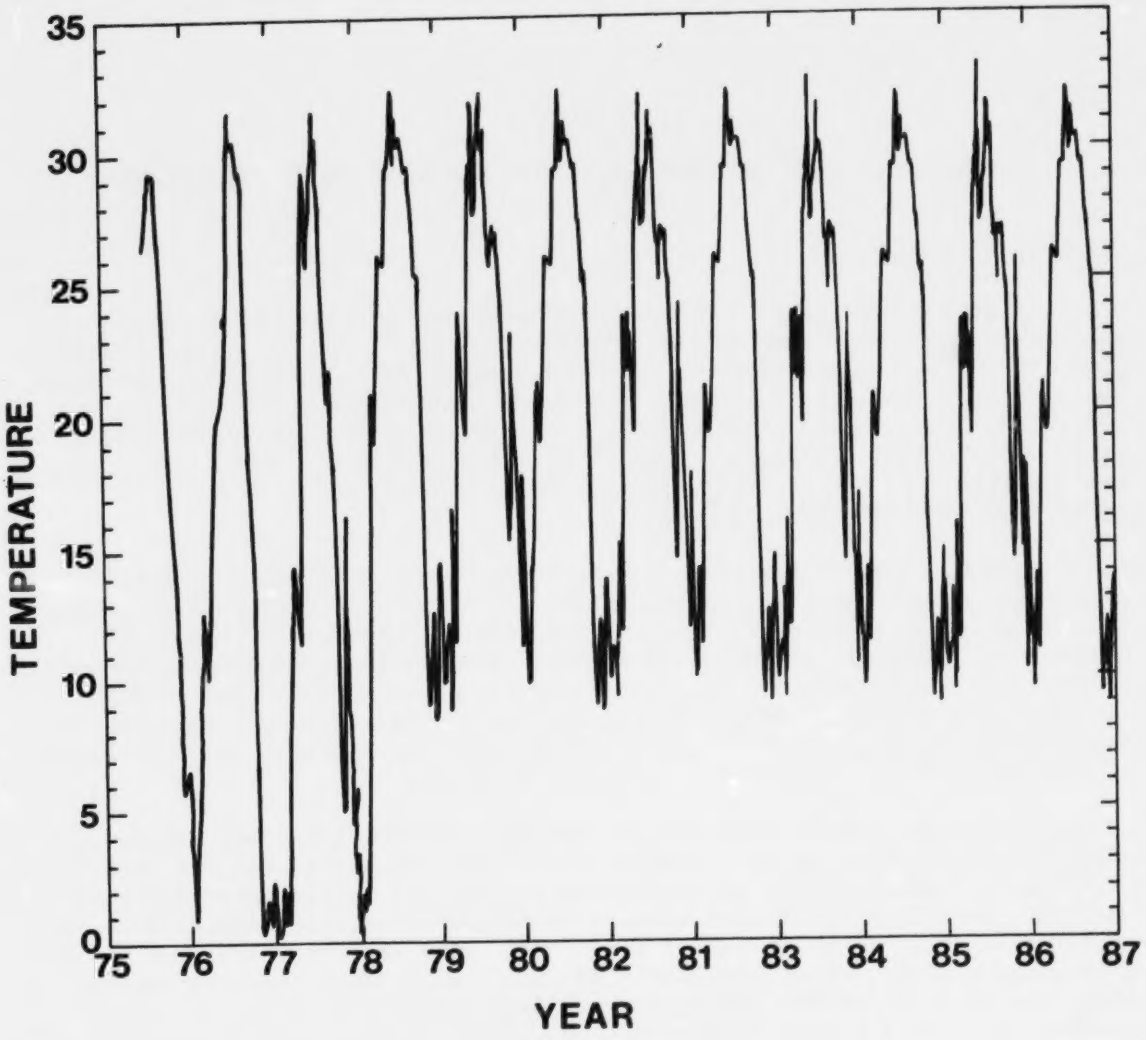

Figure 21. Cooling-lake inlet temperatures for 1975-87 used in the simulations of long-term temperature change in the ground-water system. Actual temperatures are shown from May 1975 to March 1978. Temperatures from March 1978 to January 1987 were synthesized from the 1976-78 temperature record.

temperatures may deviate widely from those assumed in this report, and processes not accounted for in the model, such as substrate decomposition and flood-induced erosion, may alter ground-water flow rates. These simulations are, however, a reasonable approximation of the changes to be expected in ground-water and substrate temperatures in the vicinity of the cooling-lake.

RESULTS AND DISCUSSION

The simulation of temperatures in the ground-water system near the cooling-lake shows that temperatures in the vegetation rooting zone in the wetland will reach a new steady-state condition that is much different from the prevailing condition before the filling of the cooling-lake. The annual temperature patterns near the dike will stabilize by 1980, but at 150 m from the dike temperatures will not reach equilibrium by 1987.

Simulated temperatures in cross-section A-A' (Figure 15) 2 m from the dike from 1975 to 1987 at a depth of 3 m deviated from those at 0.6 m by less than 1.0° C (Figure 22). When pre-lake temperatures are compared with simulated temperatures at 0.6 m depth and 2 m west of the dike in cross-section A-A' for the periods 1978-86 and 1984-86, the change in annual temperature patterns is pronounced. In the period 1984-86 the temperature does not fall below 14° C, whereas before the cooling lake was filled, substrate temperatures at this depth approached freezing and in the 1976-78 period the temperature fell below 5° C (Figure 23). The peak temperatures occur approximately 45 days later in the period 1976-78 than in the pre-lake period and approximately 30 days later in the 1984-86 period than in the pre-lake period. The lag decreases as the ground-water temperature rises because the hydraulic conductivity of the subsurface material increases with temperature, which increases the ground-water flow velocity.

The simulated temperatures in cross section A-A' 150 m from the dike for the period 1975-87 are shown in Figure 24. Pre-lake temperatures at a depth of 0.6 m and temperatures simulated for the periods 1976-78 and 1985-87 are shown in Figure 25. The attenuation in the annual temperature patterns between the pre-lake period and the 1976-78 period is a result of the increased ground-water flow rate in the latter period. At a distance of 150 m from the dikes, several years are required before the impact of the temperature variations in the lake become pronounced at a depth of 0.6 m. By 1986 the change in temperatures is pronounced with maximum and minimum annual temperatures elevated several degrees centigrade above pre-lake temperatures.

In cross-section A-A' (Figure 15) most of the ground-water discharge occurs within 200 m of the dike. Consequently, the atmospheric heat flux beyond this distance is much larger than the ground-water heat flux, and thus substrate temperatures in 1987 at a depth of 0.6 m are only slightly altered from pre-lake conditions. Ground-water temperatures at greater depths do show increases, but the rate of increase at a depth of several meters is much slower than shown in Figure 24 for a point at a depth of 0.6 m and a distance of 150 m from the dike. (Text continues on p. 48.)

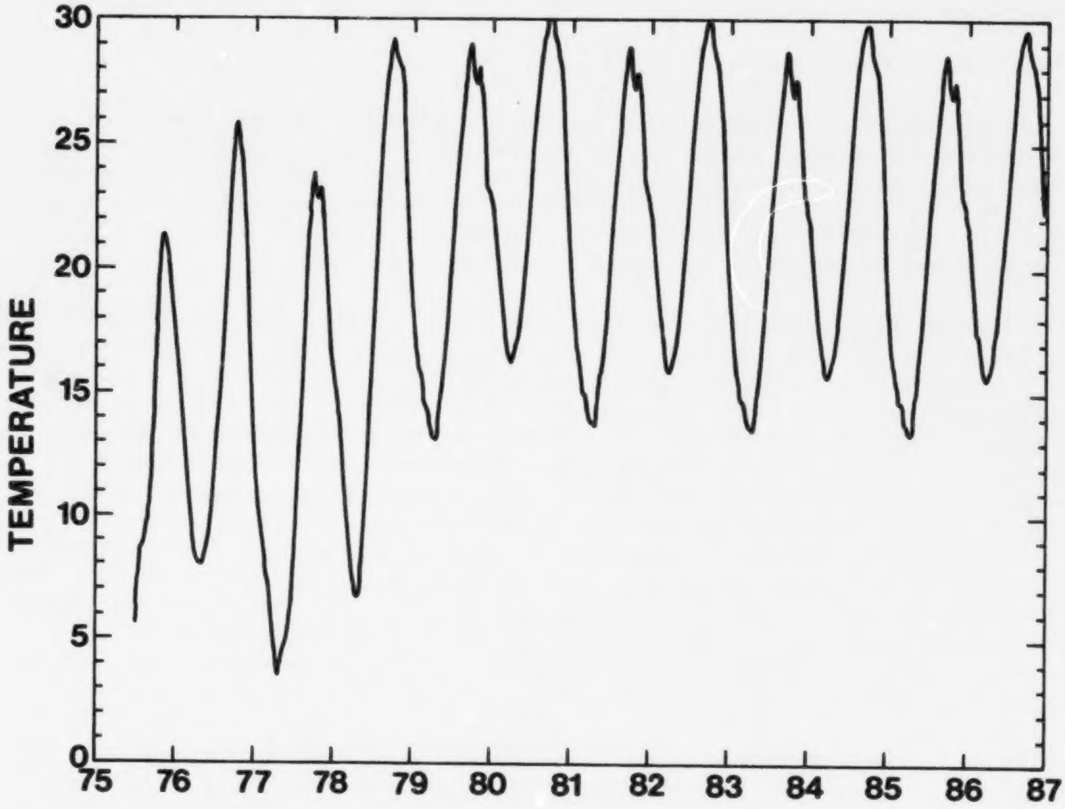

Figure 22. Predicted ground-water temperatures from 1975-87 at a distance of 2 m west of the cooling-lake dike in cross-section A-A' of Figure 15 at a depth of 0.6 m.

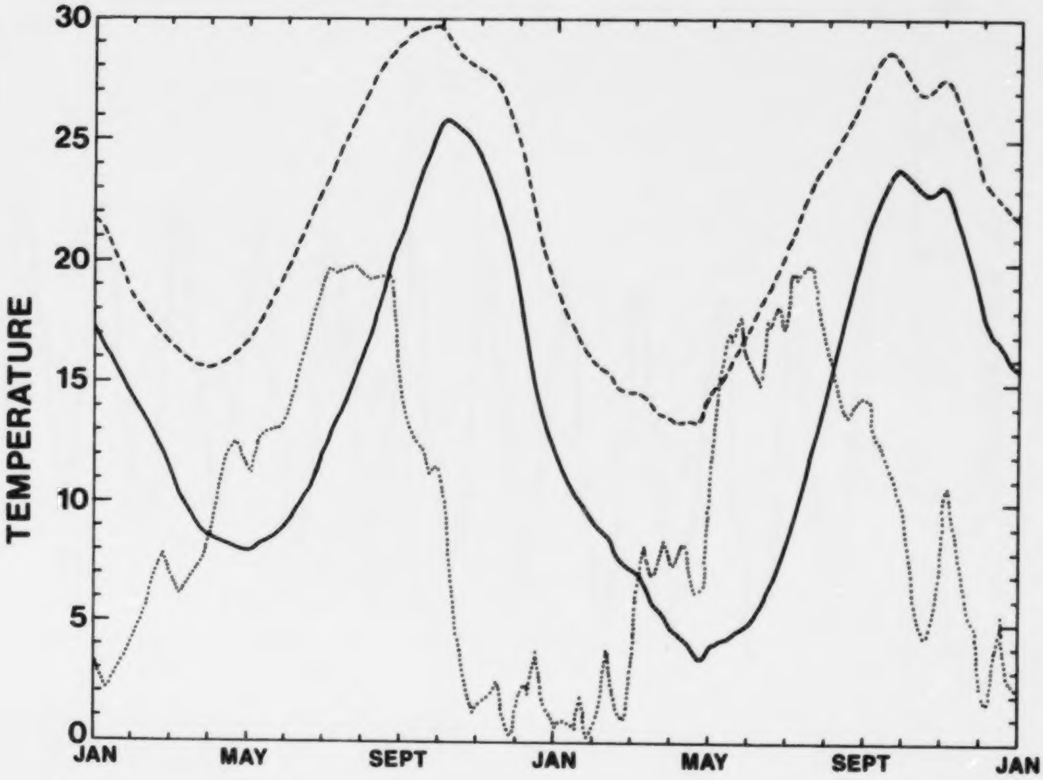

Figure 23. Pre-lake temperatures and simulated temperatures 2 m west of the cooling-lake dike in cross-section A-A' of Figure 15 at a depth of 0.6 m. (---) = simulated temperatures from 1984 to 1986; (——) = actual temperatures from 1976 to 1978; (···) = simulated temperatures from 1976 to 1978 assuming that the cooling lake had not been present.

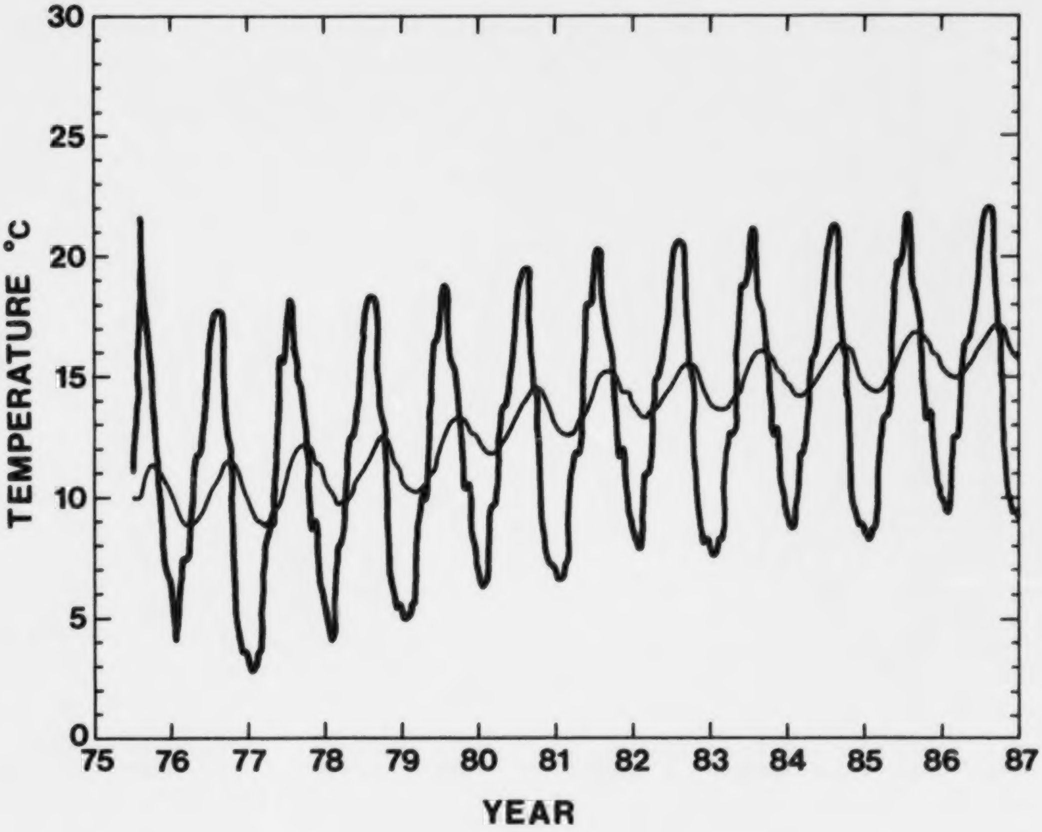

Figure 24. Temperatures 150 m west of the cooling-lake dike in cross-section A-A' of Figure 15 at depths of 0.6 m and 3 m for the simulated period 1975-87. (▬), 0.6 m level; (—), 3 m level.

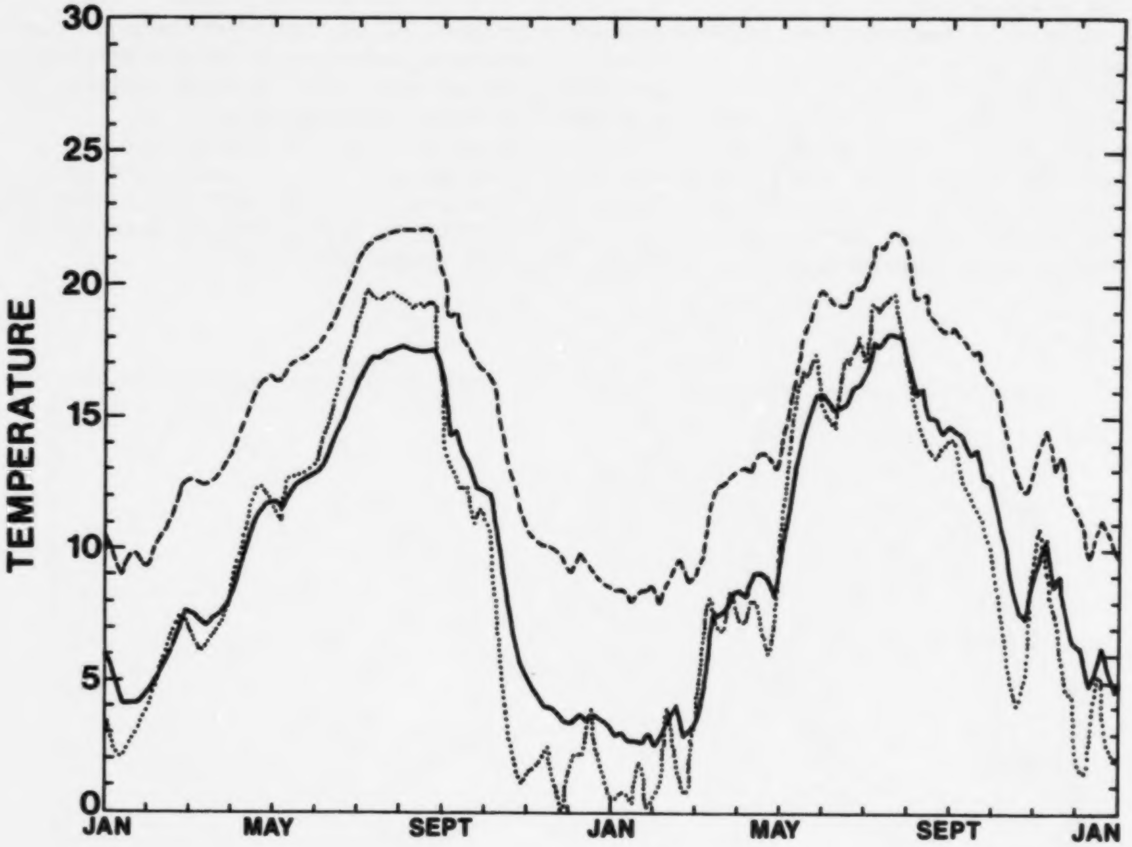

Figure 25. Temperatures 150 m west of the cooling-lake dike in cross-section A-A' of Figure 15 at a depth of 0.6 m. (---) = simulated temperatures for 1984-86; (——) = actual temperatures for 1976-78; (···) = simulated temperatures for 1976-78.

The temperature changes simulated for cross-section A-A' are believed to be representative of the changes in temperature that will occur in the vegetation rooting zone in the wetland along the dike. The ground-water flow patterns, which are determined by the subsurface stratigraphy, determine that the obvious temperature changes will occur within 350 m of the dike. The increase in temperatures will be somewhat greater in the southern part of the wetland since the average annual lake temperature increases from approximately 21° C at the intake to 24° C at the south end of the lake. This difference is shown in Figure 26 (a-c), which presents simulated temperatures in cross-section B-B' of Figure 15. This cross section is 200 m south of the cooling lake intake and 1,500 m south of cross-section A-A'. Temperatures were simulated for 1985-87 at a depth of 0.6 m and at 2, 50, and 200 m west of the dike. Temperatures 2 m west of the dike (Figure 26a) averaged about 3° C higher than temperatures at a similar point on cross-section A-A'.

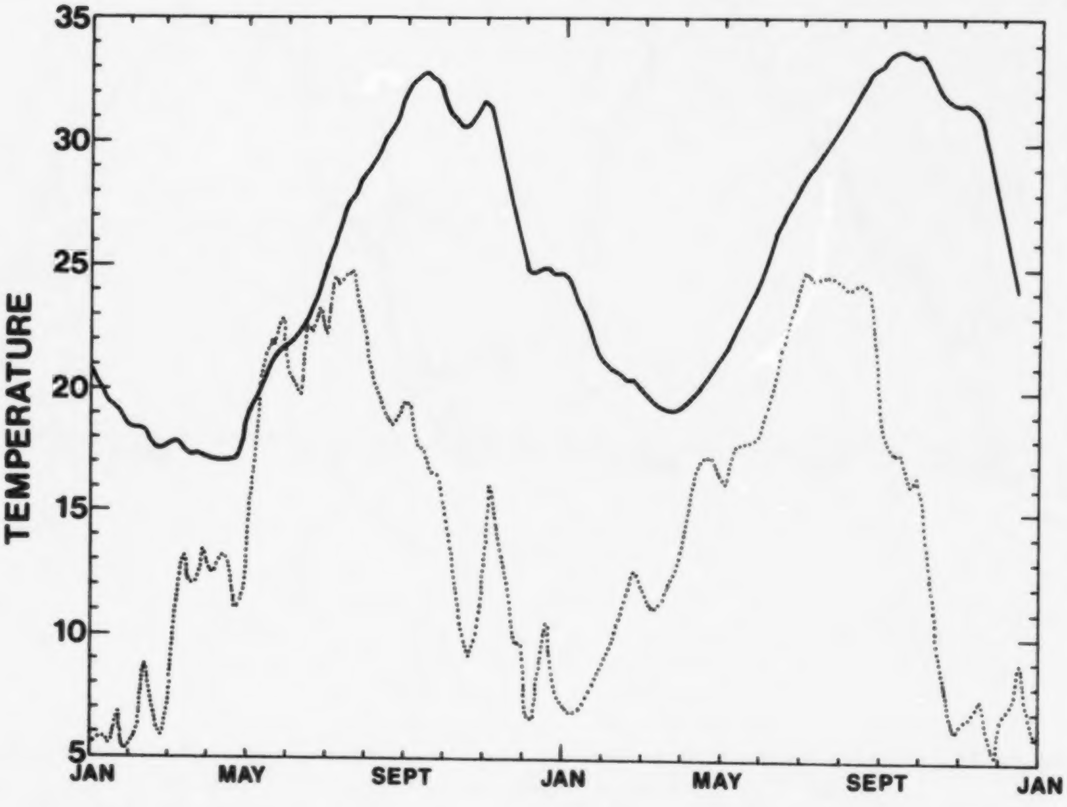

Figure 26a. Temperatures in cross-section B-B' of Figure 15 at a depth of 0.6 m and at 2 m west of the cooling-lake dike. (——) = simulated temperatures for 1985-87; (···) = simulated temperatures for 1976-78 assuming no cooling lake was present.

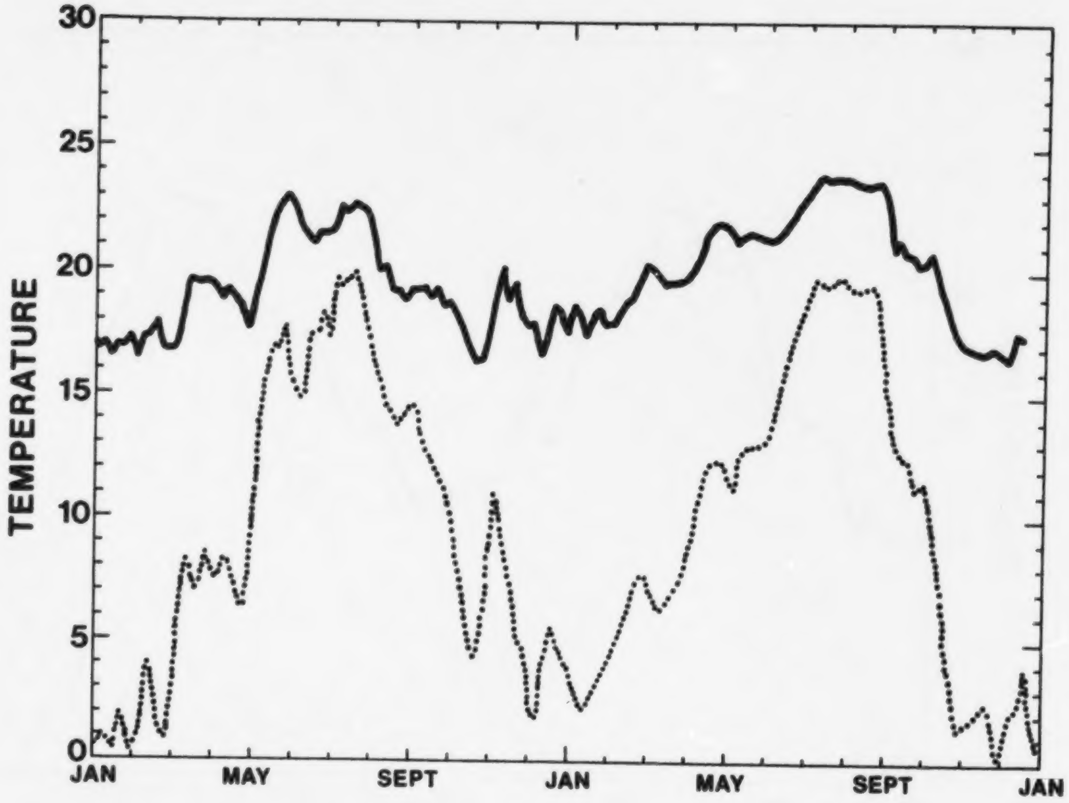

Figure 26b. Temperatures in cross-section B-B' of Figure 15 at a depth of 0.6 m and at 50 m west of the cooling-lake dike. (——) = simulated temperatures for 1985-87; (···) = simulated temperatures for 1976-78 assuming no cooling lake was present.

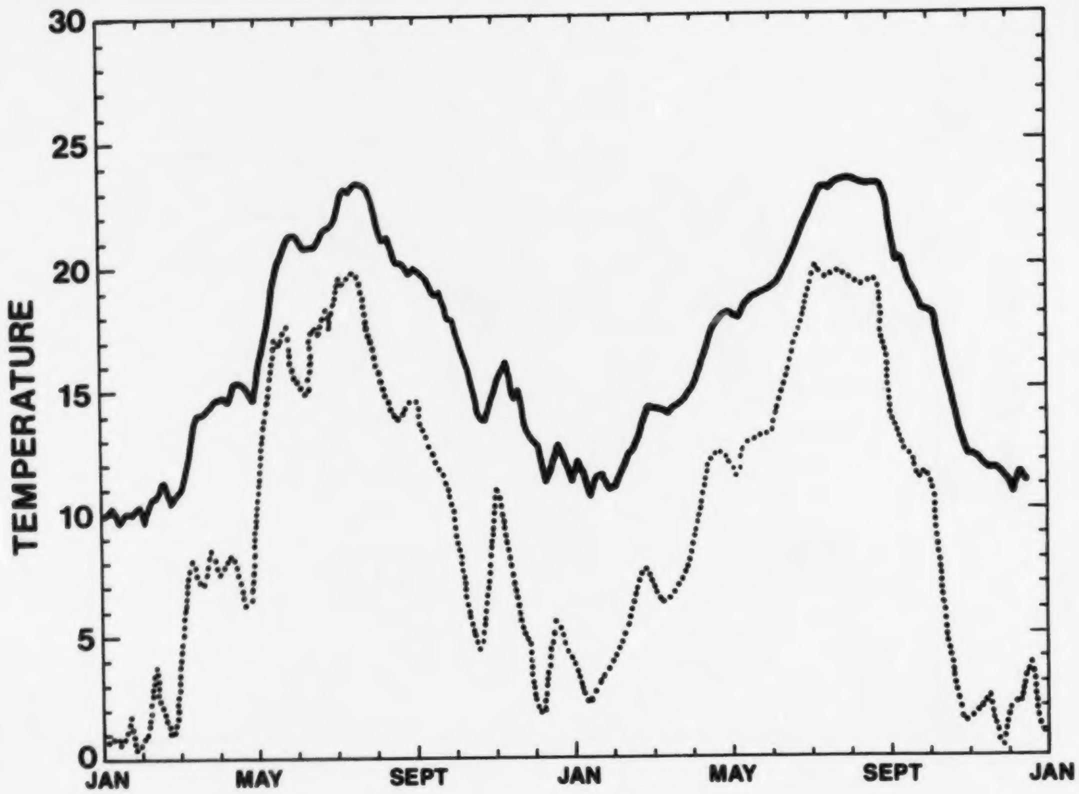

Figure 26c. Temperatures in cross-section B-B' of Figure 15 at a depth of 0.6 m and at 200 m west of the cooling-lake dike. (——) = simulated temperatures for 1985-87; (···) = simulated temperatures for 1976-78 assuming no cooling lake was present.

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APPENDIX A

DATA SPECIFICATIONS FOR THE WATER-FLOW MODEL

This appendix details procedures used at the site of the Columbia Generating Station to gather field data on the hydrogeologic system. The data were collected from 1971 to 1975. They were used to document changes in the system caused by construction and operation of the generating station and to establish the boundary conditions and parameters for the model of the ground-water flow system.

DATA FOR SPECIFYING BOUNDARIES

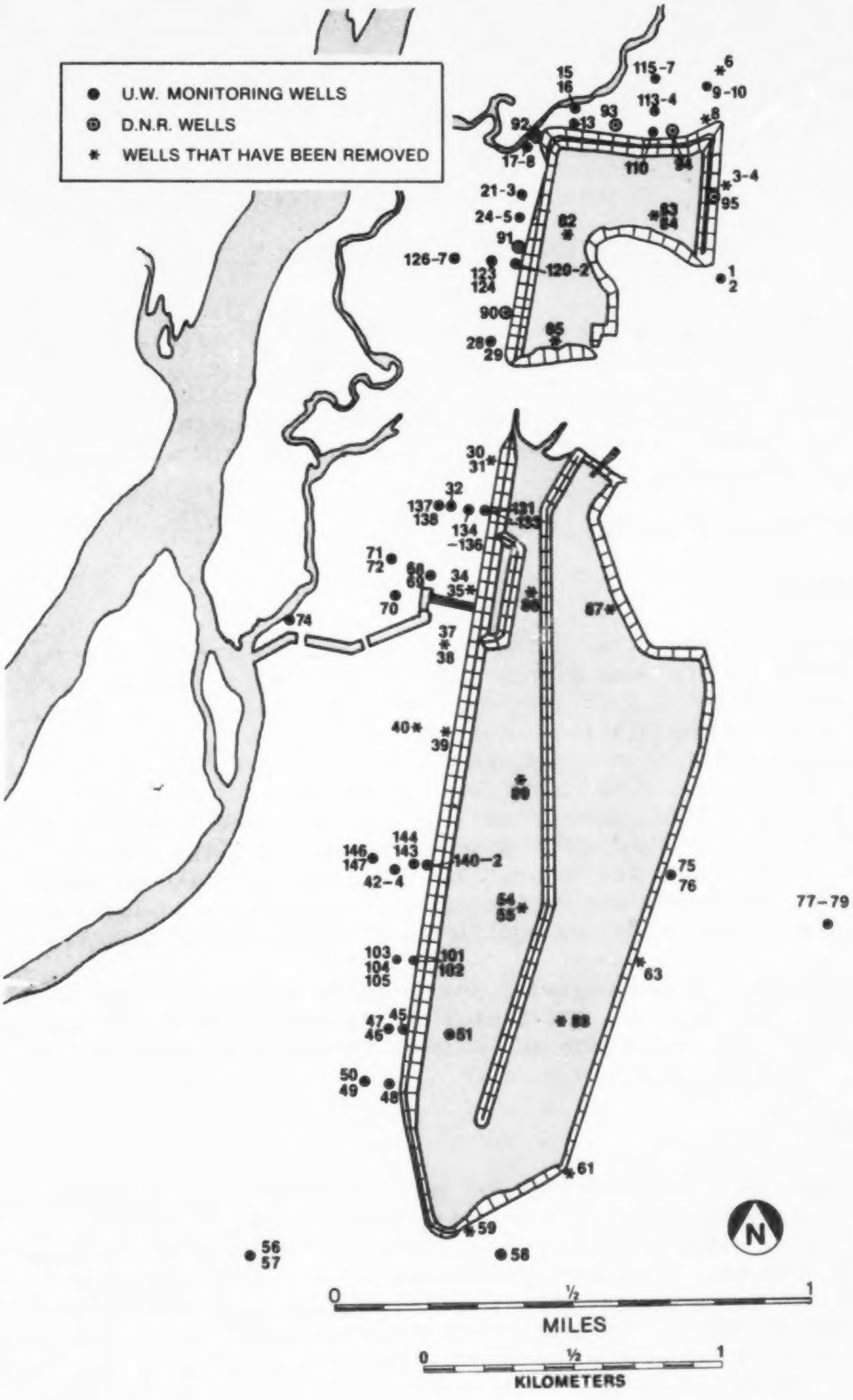
Side Boundaries

The position of the site in the regional ground-water flow system was needed to determine the size of the area to be modeled. Water-level data and well logs were collected from more than 100 domestic wells in the vicinity of the Columbia Generating station. Most of the data were obtained from the U.S. Geological Survey in Madison, Wis. The data were used to estimate aquifer transmissivity, flow rates, and the extent of the regional ground-water flow system. Transmissivity was estimated from pump tests and specific capacity data. Flow rates were computed by using the estimated transmissivity value and the water-table gradients existing in the field. The extent of the flow system was estimated by contouring water-level data from the wells open to the sandstone aquifer in the vicinity of the site.

The information on the regional ground-water system was used to justify the assumption that the left and bottom boundaries of the cross section modeled were no-flow boundaries and to specify the flow rates for the right boundary of the cross sections modeled.

Upper Boundary

Over 80 well point piezometers 3.2 cm in diameter were installed on the site to monitor the position of the water table. The piezometers had 46-cm, 80-gauge screens. They were driven in place in the lowlands and augered into place in the uplands. They were positioned so that they were open to the aquifer just below the water table. Monitoring was done with a steel measuring tape at monthly intervals beginning in summer 1971. The water levels in two wells were recorded continuously. Beginning in fall 1974 water levels were recorded at 14 points in the wetlands on a monthly basis. All observation points were surveyed to establish relative elevations. The locations of the wells are shown in Figure A-1.



● U.W. MONITORING WELLS
⊙ D.N.R. WELLS
* WELLS THAT HAVE BEEN REMOVED
15
16
115-7
6
9-10
113-4
93
92
13
8
17-8
110
94
3-4
95
21-3
24-5
82
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84
91
126-7
123
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120-2
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MILES
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½
1
KILOMETERS

Figure A-1. Location of monitoring wells at the Columbia site.

Water-level data from all monitoring wells were punched onto IBM Cards. A computer program was written in PL/1 (Programming Language, University of Maryland) that graphs the water level in each well by water year, computes the rate of change in water levels, and computes the water-table gradient between the observation points and the rate of change in the gradients. These water-level data were used to specify ground-water potential along the upper boundary of the cross sections modeled and to justify the use of a steady-state model.

DATA FOR SPECIFYING STATE VARIABLES

Permeabilities

The range of permeabilities for the various materials on the site was determined by combining data from a pump test, slug tests, and laboratory permeability tests with lithologic and grain-size information. Permeability distribution in the system was then modeled by using lithologic and grain-size information from over 100 borings on the site. (The locations of the borings are shown in appendix B.) A pump test was run in the alluvial materials on the site in winter 1972 by the Lane Engineering Firm of Chicago, Ill. The data were analyzed by using the Dupuit-Theim equation for steady radial flow without vertical movement. Slug tests were run on 23 of the observation wells on the site. Water was withdrawn from the wells, and the recovery time was recorded. The data from these tests were analyzed with the techniques developed by Cooper, Bredehoeft, and Papadopulos (1965).

Laboratory permeabilities were run on 23 samples with soil test permeameters under both falling and constant head conditions and under various density conditions. Grain-size analyses (percentages of medium sand, fine sand, silt, and clay) were run on 43 diverse samples from the site. The information was then correlated with the permeability tests, and a working model was formulated between lithology and permeability. This correlation of permeabilities with lithologies was used with the logs of the borings to develop seven cross-sectional permeability models.

Temperature

Temperatures were monitored bimonthly beginning in 1973 by lowering a Yellow Springs Instrument thermistor into observation wells. Before the temperature was recorded, the observation well was pumped with a hand-operated vacuum pump until the temperature of the water became nearly constant.

CALIBRATION DATA

In addition to the data collected to define the boundary and state variables in the system, extensive field data were collected to provide a check on model simulations.

Piezometer Nests

Nineteen observation wells were installed in the lowlands at the site. They were open between 4.5 and 7.6 m below the water table. Seven wells were installed in the uplands so that they were open between 9.1 and 18.3 m below the water table. These wells were placed adjacent to an observation well open just below the water table. Water levels in these wells were monitored at monthly intervals. The vertical gradients and the rate of change of vertical gradients have been computed for each of these wells for each monitoring date. The data from these wells were used to judge the fit of the models' simulations of the ground-water system to the actual situation.

Direct Measurements of Flow

Surface water in the wetland west of the cooling lake drains through well-defined channels during low river stages. The amount of discharge through these channels was monitored several times during fall 1975 on overcast days with a rod-suspended pygmy current meter. Measurements were taken at 0.6 of the total depth at 1-ft intervals across the streams. By similar methods the flow in the drainage ditch on the east side of the cooling lake was measured several times at both the northeast corner and the southwest corner of the cooling lake.

Discharge into the drainage ditch on the east side of the lake from the ground-water system was monitored with seepage collectors. Seepage collectors are 55-gal barrels cut in half whose open end is positioned into the substrate at the bottom of the ditch (Lee 1977). On the closed end, which is also submerged, the spout is covered by a plastic bag. The flow of water into the plastic bag is recorded, which provides a direct measurement of the ground-water inflow into the ditch in the area covered by the barrel. By means of the seepage collectors, rates of ground-water flow into the drainage ditch on the east side of the lake were determined at various locations on several different dates.

APPENDIX B

TECHNIQUES FOR DETERMINING BOUNDARY CONDITIONS AND PARAMETERS IN THE HEAT-FLOW MODEL

The model developed to simulate the transport of heat away from the cooling lake at the Columbia Generating Station requires for its solution the specification of initial and boundary conditions for the partial differential equations describing heat flow and the equations describing water flow. Five sets of parameters are also required: Hydraulic conductivity, storage coefficient, thermal conductivity, heat capacity, and dispersivity. The field and laboratory techniques used to define the boundary conditions and the parameters needed for the water-flow equation were described in appendix A. The field techniques used to measure temperature and the laboratory techniques used to measure thermal conductivity and heat capacity are described in this appendix. The technique used for specifying the dispersivity parameters is described in section 6.

TEMPERATURE MEASUREMENTS

Temperatures were recorded weekly at 40-110 points in the subsurface in the vicinity of the Columbia Generating Station site from August 1976 to January 1978 (Figure B-1). The depths at which temperatures were taken, the dates when readings were taken, and the manner in which temperatures were measured are listed in Table B-1.

Most temperatures in the field were measured with an electronic device with a thermistor as the temperature sensor. A thermistor sensor system was used because of its accuracy and its versatility. A relative accuracy of 0.1°C is easy to maintain in the field, and probes could be buried, lowered down wells, or located far from an accessible point. Liquid expansion thermographs with an accuracy of + 0.5°C were used for continuous temperature recording because these mechanical devices are much easier to maintain and less expensive than a continuous recording device with a thermistor sensor.

An electronic temperature measuring device is extremely simple in concept (Figure B-2). It consists of a thermistor whose electrical resistance is almost entirely a function of its temperature as the sensing device and an electronic measuring device to measure either directly or indirectly the resistance of the sensor, generally an ohmmeter or a wheatstone bridge. Two types of thermistors and three measuring devices were used at the Columbia Generating Station site.

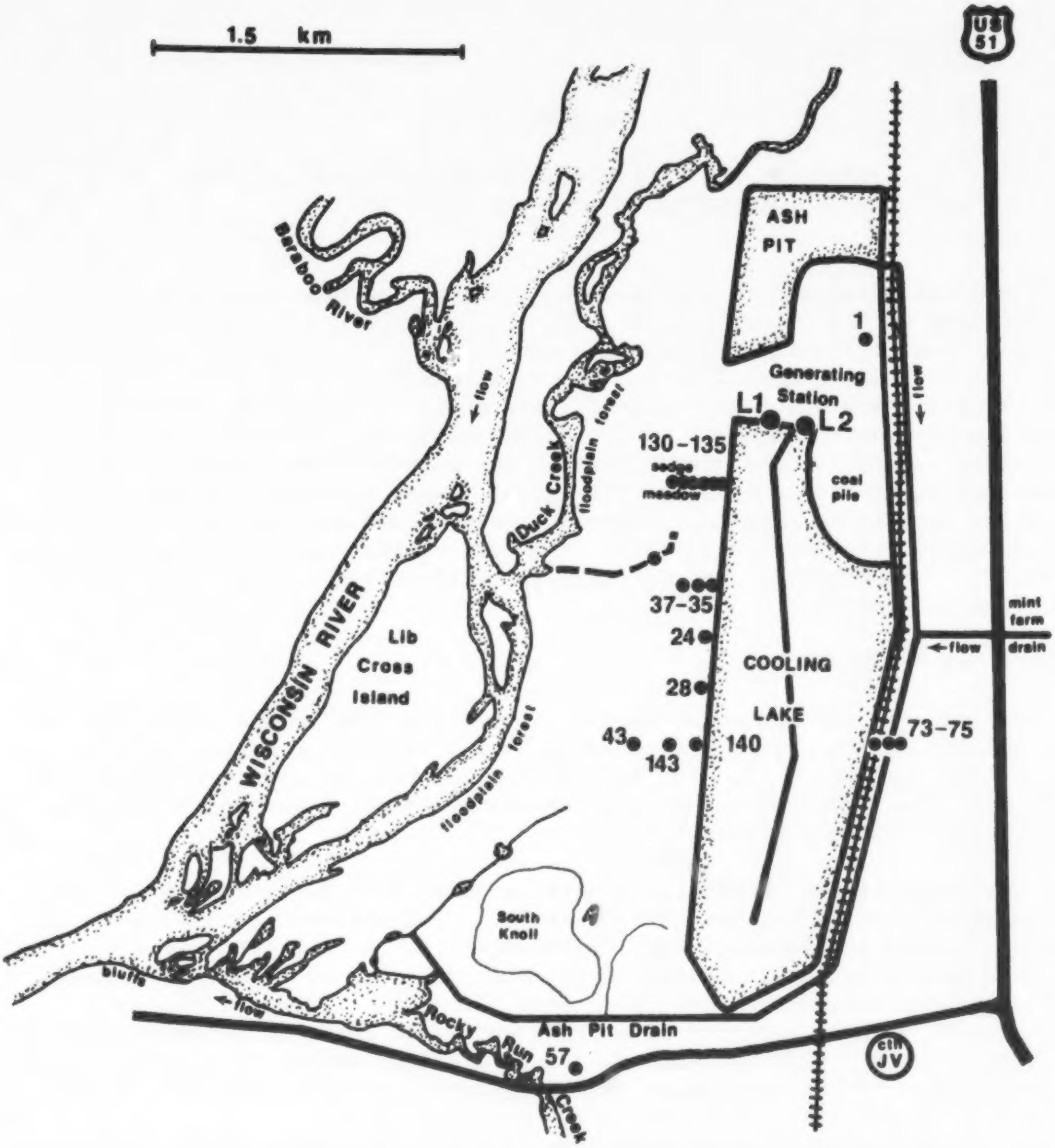

1.5 km
US 51
ASH PIT
1
Generating Station
L1
L2
flow
130-135
sedge meadow
coal pile
Baraboo River
flow
Duck Creek
floodplain forest
WISCONSIN RIVER
Lib Cross Island
37-35
24
28
COOLING LAKE
43
143
140
73-75
mint farm drain
flow
floodplain forest
South Knoll
bluffs
flow
Rocky Run
Ash Pit Drain
57
Creek
cth JV

Figure B-1. Location of temperature sampling points at the Columbia Generating Station site.

TABLE B-1. TEMPERATURE SAMPLING POINTS: LOCATIONS, DEPTHS, PROBES, AND FREQUENCIES OF READINGS

Sampling point number	Distance from dike (m)	Depth below surface at which temperature was measured (m)	Probe used to take temperature	Approximate frequency of readings
130	2	0.15, 0.91, 1.52, 3.05, 6.10, 8.70	*in situ* probe	8/8/76-8/77 weekly 9/77-1/78 monthly
131	15	0.15, 0.46, 0.91 1.51, 3.05, 4.57 6.10, 9.76	*in situ* probe	8/8/76-8/77 weekly 9/77-1/78 monthly
132	32.5	0.15, 0.46, 0.91 1.52, 3.05, 4.57 6.10, 9.76	*in situ* probe	8/8/76-8/77 weekly 9/77-1/78 monthly
133	51		*in situ* probe	8/8/76-8/77 weekly 9/77-1/78 monthly
134	86		*in situ* probe	8/8/76-8/77 weekly 9/77-1/78 monthly
135	131.5		*in situ* probe	8/8/76-8/77 weekly 9/77-1/78 monthly
35	3	2,3,4,5,6,7,8,9	probe lowered down 3.175 cm; PVC pipe	8/8/76-1/17/78 weekly
36	31	2,4,6,7,8	probe lowered down 3.175 cm; PVC pipe	8/10/77-1/18/78 weekly
37	61	2,3,4,5	probe lowered down 3.175 cm; PVC pipe	8/10/77-1/18/78 weekly
68	194	4,6,8,10,12	probe lowered down 3.175 cm; galvanized pipe	8/10/77-1/18/78 weekly
24	2	3,4,5,6,7,8,9,10	probe lowered down 3.175 cm; PVC pipe	5/31/77-1/18/78 weekly
28	2	2,4,6,8,10,12,13	probe lowered down 3.175 cm; PVC pipe	8/10/77-1/18/78 weekly
140	3	2,4,6,8	probe lowered down 3.175 cm; galvanized pipe	4/22/77-1/18/78 weekly

TABLE B-1 (continued).

Sampling point number	Distance from dike (m)	Depth below surface at which temperature was measured (m)	Probe used to take temperature	Approximate frequency of readings
143	37	2,4,6,8	probe lowered down 3.175 cm; galvanized pipe	4/22/77-1/18/78 weekly
43	126	2,4,6,8,10	probe lowered down 3.175 cm; galvanized pipe	4/22/77-1/18/78 weekly
73	west side of ditch	1,2,3,4,5,6,7,8,9	probe lowered down 2.54 cm; PVC pipe	5/31/77-1/18/78 weekly
74	east side of ditch	1,2,3,4	probe lowered down 3.175 cm; PVC pipe	5/31/77-1/18/78 weekly
75	14 m east of ditch	4,6,8,10,12,16 18,20.21	probe lowered down 3.175 cm; galvanized pipe	5/31/77-1/18/78 weekly
1		10,12,14,16,18,20	probe lowered down 3.175 cm; galvanized pipe	10/1/76-12/12/78 monthly
57		8,10,12,14	probe lowered down 3.175 cm; galvanized pipe	10/1/76-12/12/78 monthly
L1/L2	Lake inlet and outlet		thermistor	daily average value recorded every day plant is operating
Areal coverage of marsh (not shown in Figure B-1)		0.1, 0.9 m at 45 locations in marsh	*in situ*	10/28/76, 11/6/76 11/13/76, 12/4/76
Transect 1 to transect 42 (not shown on Figure B-1)	0.2 m	1.0, 3.0 m	shallow temperature probe	9/30/77, 10/23/77, 12/03/77, 1/10/78, 3/30/78

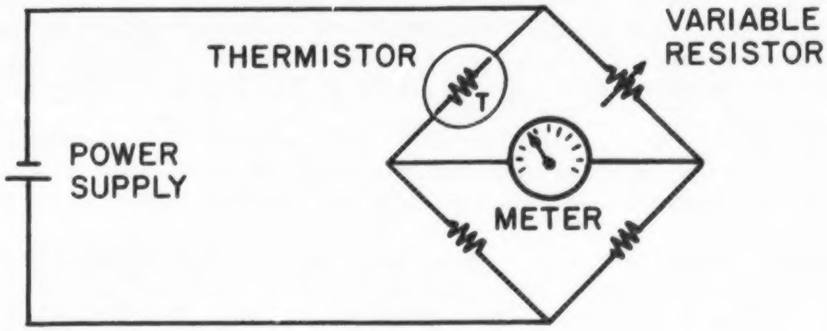

THERMISTOR
T
VARIABLE RESISTOR
POWER SUPPLY
METER

Figure B-2. A simple circuit for measuring temperature.

Thermistors

Most of the temperature data at the Columbia Generating Station site were taken with Yellow Springs Instrument Company Series 400 thermistors. These thermistors are calibrated by Yellow Springs, and all thermistors of this type have the same calibration curve with a maximum error of 0.1°C. The probes are guaranteed to remain calibrated for 1 yr. The temperature-resistance characteristics of these probes, as well as the resistance change for a 0.1° C temperature change, are shown in Figure B-3. The thermistors were used in three modes: as sensors that were lowered down a well, as sensors mounted on 2-m brass rods, and as sensors placed semipermanently in the subsurface.

The easiest and least expensive means for measuring temperature in the subsurface is to lower a probe down a fluid-filled well and take readings at various depths on the way down. The only problem with this technique is that the temperature profile in the well will not correspond to the temperature profile in the subsurface materials if the temperature gradients are above a critical value. For a 3.175-cm well the critical gradient is approximately 1°C/m at 10°C and 0.2°C at 25°C (Sammel 1968). The critical gradient was exceeded in many cases at the Columbia site. The error induced in the temperature readings is probably less than ± 0.5°C if the critical gradient is exceeded by less than 1 order of magnitude. Near the surface, where the gradient can be large, the induced error was greater. Nevertheless, because of the advantages of simply lowering a probe down a well, this technique was used to record temperatures at many locations in the subsurface.

The thermistors that were lowered down wells were purchased as a unit designed for this purpose. These thermistors were sealed by the manufacturer in an epoxy resin and attached to a shielded cable. The maximum diameter of the assembly was 0.4 cm. The time constant of these thermistors is 7 s. A thermistor assembly as described above was mounted inside a 0.95-cm brass rod, 2.1 m long, for use as a shallow temperature probe for accurately measuring temperatures near the surface. The probe could easily be pushed into the soft sediments on the Columbia site to a depth of 2 m. The thermistors purchased as a unit were calibrated with respect to each other so that temperatures between probes could be compared with an accuracy of ± 0.05°C.

In one plane perpendicular to the dike (Figure B-1), 42 thermistors were mounted in six wells with six to eight thermistors in each well. These thermistors were purchased as unmounted thermistors. Each thermistor was soldered to three wires on a cable, two wires were soldered to one lead, and the assembly was potted with silicon sealer inside a piece of tygon tubing 0.64 cm by 5 cm (Figure B-4). The wires soldered to each thermistor were of sufficient length to reach the surface. All the thermistors to be placed in a well were bound together and wired to a connector that was placed at the surface when the thermistors were inserted in the well. A cable was wired from the thermistors at each of the six wells to a control box on the dike. The control box on the dike was equipped with a 48-position switch to allow access to each thermistor. Three wires were connected to each thermistor so that the lead-wire resistance could be measured and subtracted from the resistance measured for the thermistor plus lead wire.

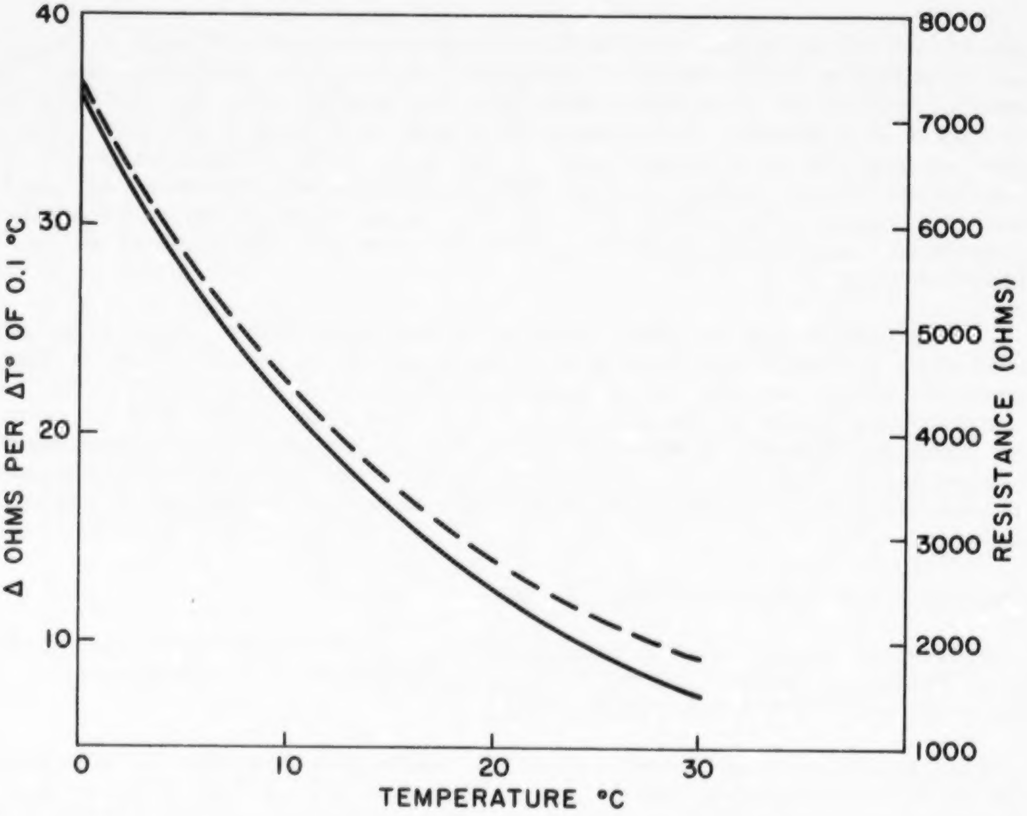


Figure B-3. Temperature-resistance characteristics of Yellow Springs Series 400 thermistors (—— = right axis) and the change in resistance with a 0.1° C change in temperature (--- = left axis).

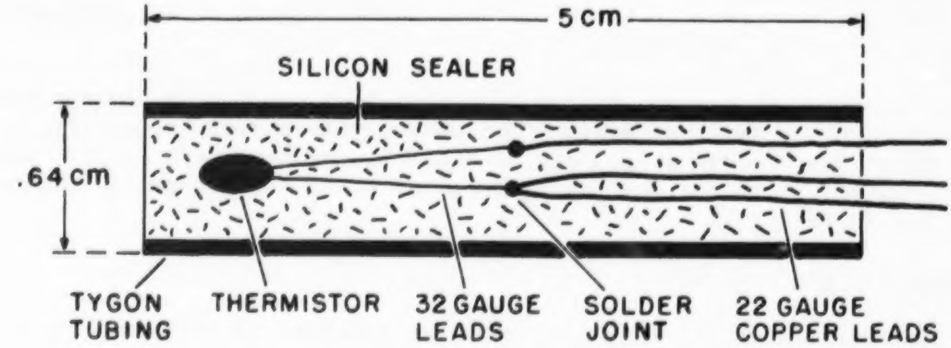

5 cm
SILICON SEALER
.64 cm
TYGON TUBING
THERMISTOR
32 GAUGE LEADS
SOLDER JOINT
22 GAUGE COPPER LEADS

Figure B-4. Potted thermistor assembly showing the details of the thermistor mounts.

The wells into which the thermistors were placed were 10-m deep, water-filled wells constructed with 3.175-cm schedule-40 PVC pipe jetted into place to minimize disturbance of subsurface materials. After the cables were placed in the wells, Fiberglas insulation was stuffed into the remaining space to minimize convection. Thermistors at depths of 0.1, 0.4, and 0.8 m were installed near the well rather than in the well. The field arrangement is shown schematically in Figure B-5. The thermistors on each cable were not calibrated relative to each other, but the cables were pulled twice during the 18 months of temperature-data collection to check for deviation from the calibrated range.

In addition to the *in situ* thermistors described above, 200 additional thermistors (Fenwall Disc Type JB31J1) were placed at 60 locations in 12 lines perpendicular to the dike in the marsh at depths of 0.1 and 0.9 m. Each line of thermistors was wired to a control box on the dike. Four sets of readings were taken in October and November 1976, but in December the multicolored 22-gauge copper wire, which was used to connect the thermistors to the control boxes on the dikes, became incorporated in the winter dwellings of the marsh muskrats.

Meters Used for Temperature Monitoring

Because of delays in obtaining equipment, three meters were used during the 17 months of temperature monitoring. All meters were frequently calibrated against a known resistance.

From August through December 1976 all thermistor resistances were measured on a Shallcraft four-dial wheatstone bridge with a 5 A full scale needle galvanometer. Resistances could be read accurately to ± 3Ω at 4,000 Ω with this meter, and it probably had an operating accuracy of ± 10 Ω.

A Digitec digital recording ohmmeter mounted in a vehicle was used for measuring resistances from January 1977 through January 1978. Most of the resistance measurements were made with this instrument. This instrument had an operating accuracy at 4,000 Ω of approximately ± 4 Ω and was operated at all times in the field with a 12 VDC to 120 VAC square wave converter and at a temperature of at least 20°C.

A four-dial wheatstone bridge with a digital null meter was constructed as a backup for the digital ohmmeter and as a portable field meter (Figure B-6). This meter had a working accuracy at 4,000 Ω of approximately ± 5 Ω and was used primarily with the shallow temperature probe for measuring temperatures near the surface.

Temperature Measurements

Two resistances for each location were recorded in the field, a line resistance and a total resistance. The resistances were then coded, punched, cross checked for errors, and then converted to temperature by the following equation (Steinhart and Hart 1968):

$$T^{-1} = A + B \log R + C(\log R)^3 ,$$

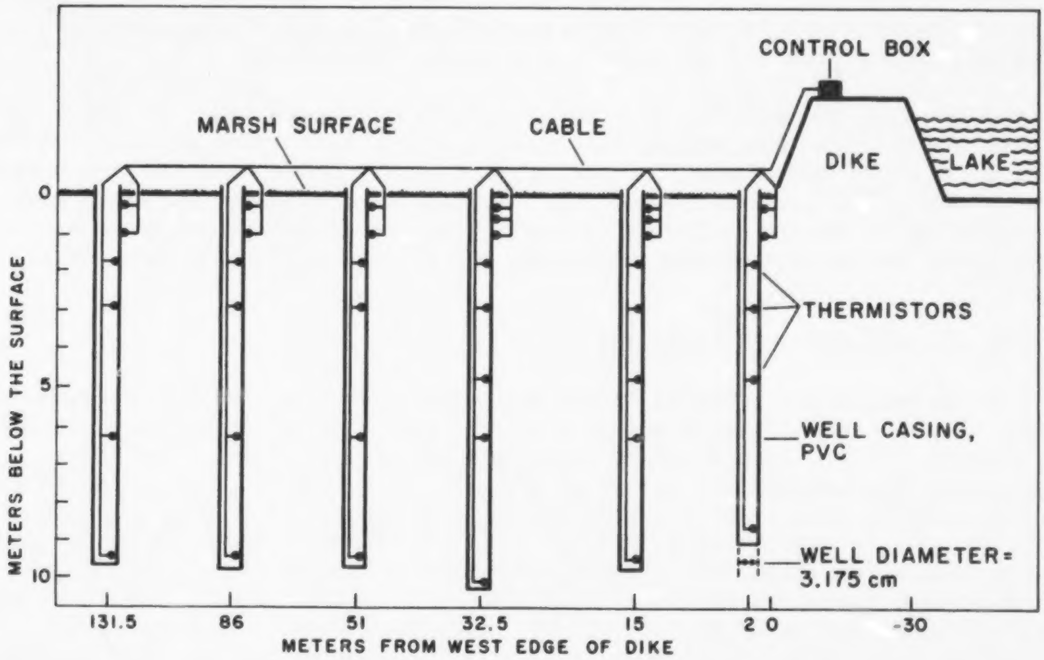

CONTROL BOX
MARSH SURFACE
CABLE
DIKE
LAKE
THERMISTORS
WELL CASING, PVC
WELL DIAMETER= 3.175 cm
METERS BELOW THE SURFACE
0
5
10
131.5
86
51
32.5
15
2
0
-30
METERS FROM WEST EDGE OF DIKE

Figure B-5. Schematic of thermistor placement in subsurface wells at the Columbia Generating Station site. The wells shown correspond to locations 130-135 of Figure B-1. Horizontal dimensions are distorted, but vertical dimensions are approximately correct.

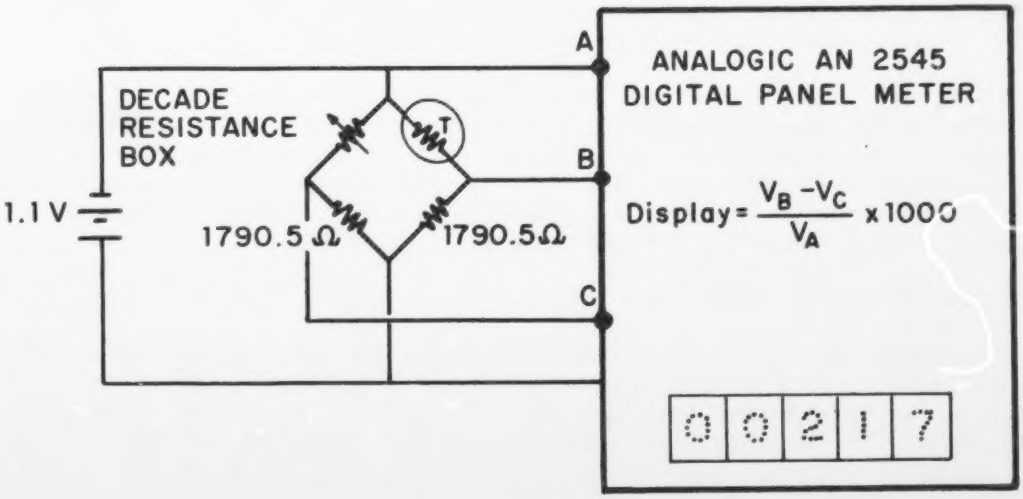

A
B
C
DECADE RESISTANCE BOX
T
1.1 V
1790.5 Ω
1790.5 Ω
ANALOGIC AN 2545 DIGITAL PANEL METER
Display = $\frac{V_B - V_C}{V_A}$ x 1000
00217

Figure B-6. Schematic of digital bridge circuit used as a portable field meter for measuring ground-water temperatures.

where the constants A, B, and C were determined from the thermistor calibration data, and T^{-1} is the inverse Kelvin temperature.

The relative accuracy of all temperature measurements taken in the study is within + 0.5°C of the actual temperature of the thermistor probe. The main sources of error were thermistor calibration, meter drift, and incorrect meter calibration. The correspondence of probe temperatures to actual subsurface temperatures is unknown. The deviation between these temperatures is a function of the mode of measurement used and was assumed to be small in all cases.

THERMAL CONDUCTIVITY DETERMINATION

The thermal conductivities of unconsolidated materials at the Columbia Generating Station site were measured by the needle probe method (Von Herzen and Maxwell 1959). In the needle probe method, which was developed for determining the thermal conductivity of deep sea sediments, a thin needle is inserted into a sample and is heated along its length. The sample can be as small as 3 cm in radius and 7 cm long. From a record of temperature increase in the needle over time, the thermal conductivity can be easily calculated. The probe and the experimental arrangement for measuring thermal conductivity for this study are shown in Figure B-7. The thermistor in the probe was calibrated to 0.05°C. The needle probe was not checked for accuracy against a known standard.

The theory for the needle probe has been worked out in detail by Jaeger (1958). His analysis shows that temperature increase of the probe is approximated by

$$T = \frac{q}{4K\Pi} \ln \left(\frac{4\alpha t}{1.7811\, a^2}\right), \quad t > a^2/\alpha ,$$

where α = thermal diffusivity of the sediment sample, L^2/t; a = probe radius, L; t = time, t; and q = heat input per unit length per unit time, H/tL.

A plot of T versus ln t will give a straight line, the slope of which determines K for known values of q. The data collected from measurements of the thermal conductivity of three samples from the Columbia Generating Station site are shown in Figure B-8. All samples were checked for reproducibility, the standard error was less than 3%, and in all measurements thermal conductivity was assumed to be independent of direction.

DETERMINATION OF HEAT CAPACITY

Since heat capacity of a composite material can be determined from a weighted average of the heat capacities of the individual parts in the composite, it was assumed that the heat capacity of the unconsolidated materials at the Columbia Generating Station site could be determined from the relation

$$\rho C_s = \rho C_w X_w + \rho C_{org} X_{org} + \rho C_{qtz} X_{qtz} + \rho C_{clay} X_{clay} ,$$

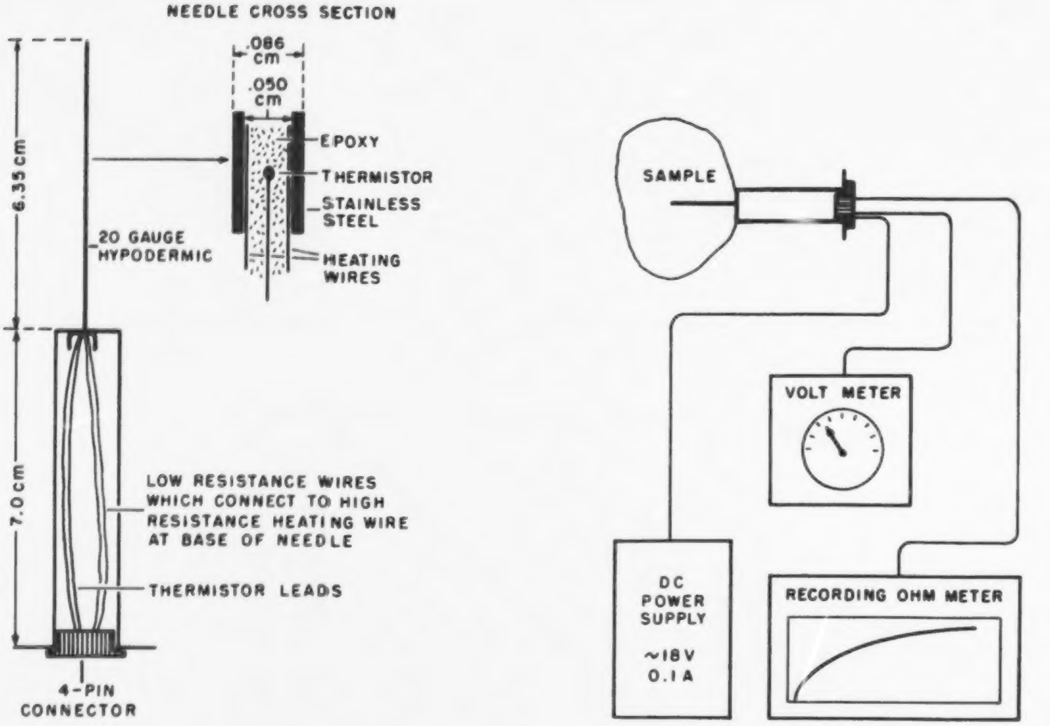

NEEDLE CROSS SECTION
.086 cm
.050 cm
EPOXY
THERMISTOR
STAINLESS STEEL
HEATING WIRES
20 GAUGE HYPODERMIC
6.35 cm
7.0 cm
LOW RESISTANCE WIRES WHICH CONNECT TO HIGH RESISTANCE HEATING WIRE AT BASE OF NEEDLE
THERMISTOR LEADS
4-PIN CONNECTOR
SAMPLE
VOLT METER
DC POWER SUPPLY
~18V
0.1A
RECORDING OHM METER

Figure B-7. Details of (a) the needle probe and (b) the experimental arrangement to measure thermal conductivities of unconsolidated materials. The power supply was used to supply a constant voltage to the needle probe. The voltage was then monitored on the volt meter, and the ohmmeter was used to record the resistance changes in the thermistor embedded in the thermistor.

where X_w, X_{org}, X_{qtz}, and X_{clay} denote fractional parts of the material made up of water, organic material, quartz and feldspar, and clay minerals, respectively. The heat capacities of these four components are listed in Table B-2. In a given sample all volatiles were assumed to be organic, all materials greater than 0.001 mm in diameter were assumed to be quartz and feldspars, and all materials less than 0.001 mm in diameter were assumed to be clay minerals. The water content of the samples was determined in the Quarternary Laboratory at the University of Wisconsin-Madison, and the organic, sand and silt, and clay fractions were determined by the Soil and Plant Laboratory of the University of Wisconsin-Madison.

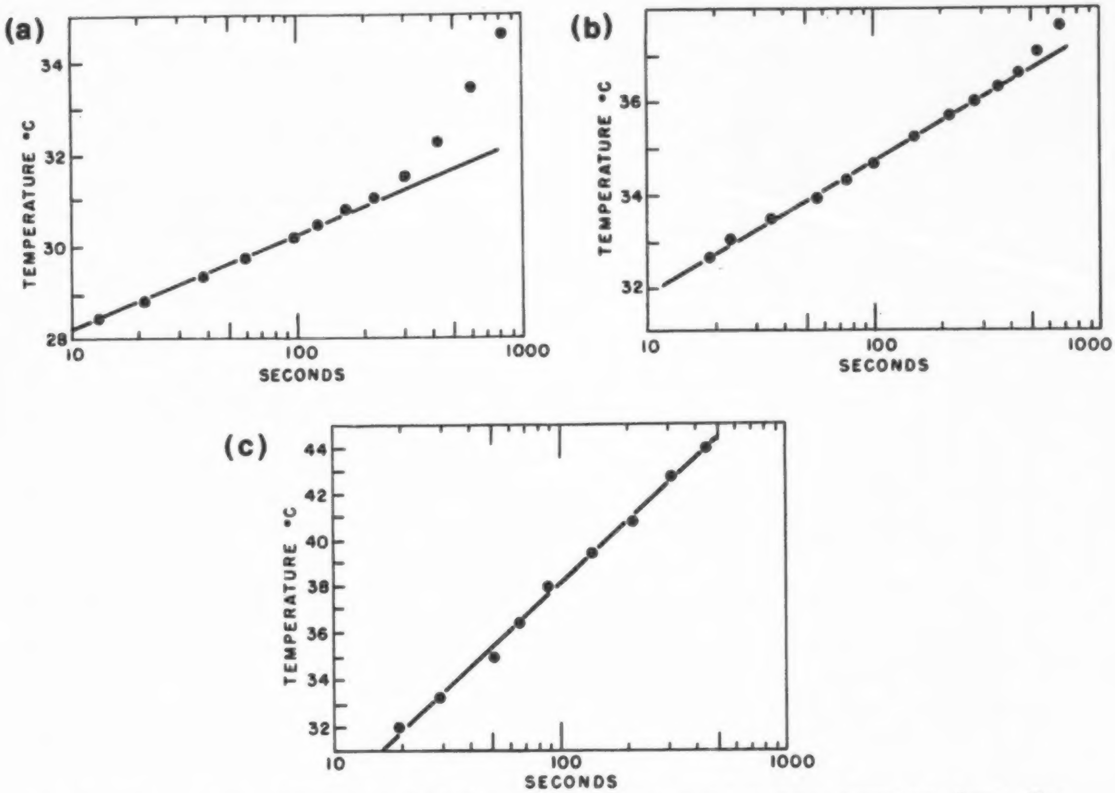


Figure B-8. Thermal conductivities of three ground-water samples from the Columbia Generating Station site which were analyzed with the needle probe. Thermal conductivities were calculated to be 4.9 x 10^{-3} cal/°C cm sec for medium to fine sand (a), 5.0 x 10^{-3} cal/°C cm sec for fine sand (b), and 1.5 x 10^{-3} cal/°C cm sec for hemic peat (c).

TABLE B-2. HEAT CAPACITIES OF THE COMMON COMPONENTS OF UNCONSOLIDATED GLACIAL MATERIALS[a]

Component	Heat capacity (cal/m^3°C)
Quartz and feldspar	0.51 x 10^6
Clay minerals	0.51 x 10^6
Organic matter	0.60 x 10^6
Water @ 25°C	1.00 x 10^6

[a] Adapted from Van Wijk and de Vries (1963).

APPENDIX C

A FINITE ELEMENT PROGRAM TO SIMULATE SINGLE-PHASE HEAT FLOW OR CONSERVATIVE MASS TRANSPORT IN AN AQUIFER

BRIEF DESCRIPTION OF THE MODEL

The following computer code is designed to solve the two-dimensional equations of ground-water flow and heat or mass transport for an aquifer by using the finite element method. The program was written to facilitate the analysis of ground-water contamination in shallow glacial aquifers. The program solves for aquifer potentials and temperatures or concentrations. However, simultaneous simulation of the transfer of heat and mass in an aquifer is not possible. The aquifer to be simulated may be artesian, a water table, or a combination of both. It may be heterogeneous and anisotropic and may have irregular boundaries. The program was designed to simulate cross-sectional problems, but it has also been used successfully to simulate areal problems.

The basic procedure is to solve a ground-water flow problem for potentials in an aquifer and then to use the potential information to compute a velocity distribution in the aquifer. The heat or mass transport equation is then solved for temperatures or concentrations (Figure C-1).

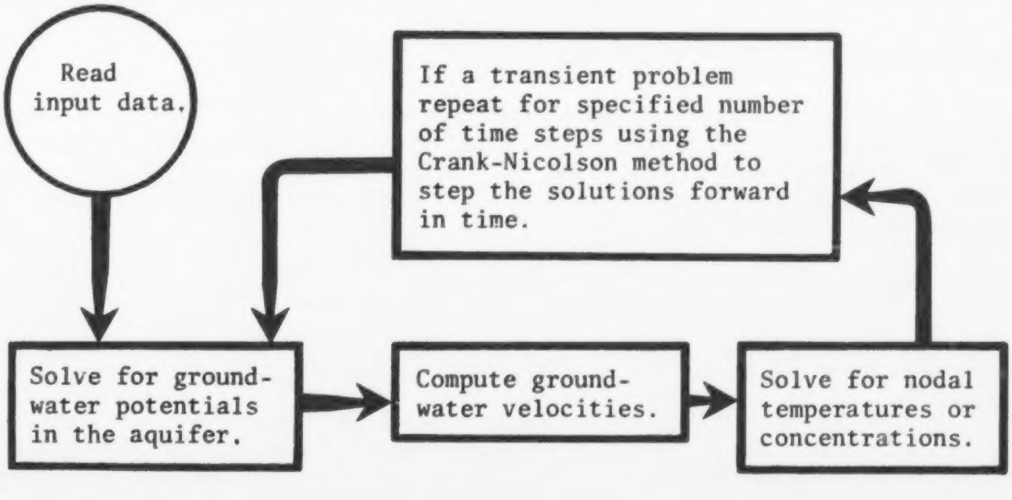

Read input data.
If a transient problem repeat for specified number of time steps using the Crank-Nicolson method to step the solutions forward in time.
Solve for ground-water potentials in the aquifer.
Compute ground-water velocities.
Solve for nodal temperatures or concentrations.

Figure C-1. The basic procedure for linking the ground-water flow and the transport equations of the model. In general, smaller time steps will be used for the heat/mass transport equation than for the water-flow equation.

This flexible program includes these features:

1) Neuman, Dirichlet, and mixed boundary conditions can be handled.

2) The basic element shape is a quadrilateral. The sides of the elements can be linear, quadratic, or cubic, and thus element sides can have two, three, or four nodes. Any given element can have four linear sides or any mix of linear, quadratic, and cubic sides. This feature allows considerable flexibility in designing a finite element grid.

3) The program handles hydrodynamic dispersion.

4) Point, line, or areal sources of water, heat, or mass can be represented.

5) The problem need not be a linked problem; if desired, the model can be used to solve only for aquifer potentials.

6) Hydraulic conductivity can be represented as a function of temperature.

The program is relatively simple to use, but those unfamiliar with the development of the equation describing the convective-dispersive transport of heat or mass in an aquifer, or those unfamiliar with the limitation of the numerical approximation technique, may experience difficulties in using the program. Oscillations and instabilities are frequently encountered in solving the convective-dispersive equation because (1) the equation behaves like a hyperbolic partial differential equation when convective transport dominates over diffusive transport; (2) in the numerical technique used in the program velocities are not continuous everywhere; and (3) the numerical scheme cannot transmit sharp contaminant fronts. Oscillations and instabilities can generally be dampened by reducing element size or by reducing the time step.

THE EQUATIONS SOLVED BY THE PROGRAM

The program was written to solve the differential equation describing mass or energy transport in a ground-water aquifer. Since the rate of transport is a function of ground-water velocity, the first step in solving the transport equation is to solve the ground-water flow equation so that ground-water velocities can be determined.

Water-Flow Equation

The following partial differential equation, which can be used to describe ground-water flow in a confined or water-table aquifer, is solved by the program

$$\frac{\partial}{\partial x_i}\left(bK_{ij}\frac{\partial \phi}{\partial x_j}\right) + W - S\frac{\partial \phi}{\partial t} = 0 , \quad i,j - 1,2 , \qquad \text{(C-1)}$$

where K_{ij} are the components of the hydraulic conductivity tensor, which may be a function of temperature, Lt^{-1}; b = aquifer thickness, L; ϕ = ground-water potential, L; W = flux of recharge per unit area, L/T; and S = storage coefficient for a confined aquifer, or the specific yield in a water-table aquifer, dimensionless. Boundary conditions may be some combination of the following:

$$\phi = \text{constant}, \tag{C-2}$$

$$\frac{\partial \phi}{\partial x_i} = \text{constant}. \tag{C-3}$$

Equation C-1 can be used only if the aquifer is assumed to be incompressible and if the linearized Dupuit-Forcheimer assumptions apply for unconfined flow for an areal problem. Pinder and Bredehoeft (1968) discuss the development of Eq. (C-1) for modeling a confined aquifer, and Bredehoeft and Pinder (1970) discuss the use of Eq. (C-1) for modeling a water-table aquifer.

Mass/Heat Transport Equation

The program solves the following partial differential equation, which can be used to model the flow of heat or mass in porous media and is commonly referred to as the convective-dispersive equation:

$$\frac{\partial}{\partial x_i} (D_{ij} \frac{\partial C}{\partial x_j}) + \frac{\partial}{\partial x_1} (g\ D'_{ij} \frac{\partial C}{\partial x_j}) - g\ q_i \frac{\partial C}{\partial x_i} - R - \rho \frac{\partial C}{\partial t} = 0\ , \tag{C-4}$$

where

$$D'_{11} = \alpha_L \frac{q_1 q_1}{q} + \alpha_T \frac{q_2 q_2}{q} \quad ;$$

$$D'_{21} = D_{12} = (\alpha_L - \alpha_T) \frac{q_1 q_2}{q} \quad ;$$

$$D'_{22} = \alpha_L \frac{q_2 q_2}{q} + \alpha_T \frac{q_1 q_1}{q} \quad ;$$

α_L = longitudinal dispersivity, L; TS = transverse dispersivity, L; q_i = velocity or specific discharge, which is calculated by

$$q_i = -K_{ij} \frac{\partial \phi}{\partial x_i} \quad ; \tag{C-5}$$

q = mean velocity, $(q_1 + q_2)^{1/2}$, L/t.

The remainder of the parameters are defined differently for a mass transport problem and for a heat transport problem (Table C-1). Boundary conditions may be some combination of the following:

$$C = \text{constant}, \tag{C-6}$$

$$\frac{\partial C}{\partial x_i} = \text{constant}, \tag{C-7}$$

$$\frac{\partial C}{\partial x_i}\ \alpha\ q_i C_i\ , \tag{C-8}$$

TABLE C-1. PARAMETERS FOR BOTH MASS AND HEAT TRANSPORT PROBLEMS

	Mass transport problem	Heat transport problem
D_{ij}	Components of molecular diffusion tensor, L^2/t;	Components of the thermal conductivity tensor for the saturated medium, H/t LT;
C	Mass concentration, M;	Temperature, T;
g	Unity, dimensionless;	Heat capacity of water, H/L^3T;
	Effective porosity, dimensionless;	Heat capacity of the saturated medium, H/L^3T;
R	Mass recharge rate, M/t.	Heat generation or recharge rate, H/L^3t.

$$\frac{\partial C}{\partial x_i} \alpha \; q_i C\infty \quad , \tag{C-9}$$

$$\frac{\partial C}{\partial x_i} \alpha \; C - C\infty \tag{C-10}$$

where C = concentration beyond an exterior boundary.

A physical significance can be ascribed to each of the terms in Eq. (C-4), since the equation can be thought of as a mass balance equation which states that flow into and out of a differential volume sums to zero. The processes described by each of the terms are: transfer by molecular diffusion,

$\frac{\partial}{\partial x_i} (D_{ij} \frac{\partial C}{\partial x_j})$; transfer by velocity fluctuations, called dispersive transfer, $\frac{\partial}{\partial x_i}(g\, D'_{ij} \frac{\partial C}{\partial x_j})$; convective transfer, $g\, q_i \frac{\partial C}{\partial x_i}$; source or sink, R: storage changes, $\rho \frac{\partial C}{\partial t}$.

The use of Eq. (C-4) to describe mass or heat transport in an aquifer implies the following assumptions: (1) the system is chemically inert, (2) the aquifer is incompressible, (3) density is constant within the system, (4) all transport occurs by molecular diffusion, mechanical dispersion, and convective transport, (5) flow can be represented in two dimensions, and (6) the flow of water is laminar. Bear (1972) and Konikow and Grove (1977) present derivations of Eq. (C-4) to describe solute transport in ground water. The development of Eq. (C-4) to describe energy transport in ground water will be discussed.

THE APPROXIMATION TECHNIQUE

The finite element method is used in the program to approximate Eq. (C-1) and Eq. (C-4) to solve for the unknown potentials and concentrations in an aquifer. The finite element method, like the finite difference method, is a discretization technique. The continuous aquifer is represented by a number of regions, called elements, each bounded by a set of nodes. Integral equations describing water flow and heat or mass flow are developed for each region, giving two sets of simultaneous algebraic equations which are solved to determine the unknown potentials and concentrations. The finite element method generates the integral equation from the governing differential equation and evaluates the integral over an element by a numerical integration scheme. Thus, the finite element method operates on a region, whereas the finite difference technique only operates on a point.

The finite element method has several advantages over the finite difference method for solving Eq. (C-1) and Eq. (C-4), and, in fact, the finite difference method may be unacceptable for solving Eq. (C-4) (Pinder 1973). The advantages are: (1) The finite element method is characterized by less overshoot and less numerical smearing of a concentration front (Pinder and Gray 1977); (2) velocities can have a functional representation, which reduces oscillations in the solution to Eq. (C-4); (3) boundary conditions are much easier to treat in the finite element method, and (4) irregular boundaries are easier to handle with the finite element method.

Two techniques can be used in formulating the approximating integral equations that are the foundation of the finite element method. The procedure used in this study to develop the required finite element formulation is the Rayleigh-Ritz procedure, or the minimum potential energy procedure, which is based on the calculus of variation. The other procedure, the Galerkin method, is more general in application and has been in favor in the literature. It was used by Pinder and Gray (1977) to develop the approximating integral equations for equations similar to Eq. (C-1) and Eq. (C-4). Both methods yield identical finite element formulations for Eq. (C-1) and Eq. (C-4).

The Finite Element Formulation

The basis of the variational principle is to develop an integral equation describing the total energy within a region and then to minimize the integral. This principle implies that potentials and concentrations in an aquifer will always be at levels that minimize total energy for a given set of boundary conditions.

Integral equations can be developed from first principles by quantifying potential energy in the system, but the Euler-Lagrange equation (Myers 1971) facilitates the process by transforming the governing partial differential equations to integral equations that describe total energy within a region.

The variational statements for Eq. (C-1) and Eq. (C-4) for an element within the aquifer are

$$\Pi_w = 1/2 \iint K_{ij}(\frac{\partial\phi}{\partial x} \frac{\partial\phi}{\partial x})\, dx_1 dx_2 - 1/2 \iint [2W\phi + S\frac{\partial\phi^2}{\partial t}]\, dx_1 dx_2 \ ; \qquad (C\text{-}11)$$

and

$$\Pi_T = 1/2 \iint [D_{ij} \ (\frac{\partial C}{\partial x_i} \frac{\partial C}{\partial x_j}) + g(D'_{ij}) \frac{\partial C}{\partial x_i} \frac{\partial C}{\partial x_j} - g\ q_i \frac{\partial C}{\partial x_i} C - 2RC$$

$$- \rho \frac{\partial C^2}{\partial t}] b dx_1 dx_2 + 1/2 \ \S \ H(C^2 - 2CC\infty) bds + 1/2 \ \S \ gq_i \ C^2 bdS$$

$$+ 2 \ \S \ gq_i \ C\infty CbdS \ ; \qquad (C\text{-}12)$$

where Π_W = total potential for water flow, Π_T = total potential for heat or mass transport, H = convection coefficient (units depend on type of problem), and b = aquifer thickness.

The next step is to minimize Eq. (C-11) and Eq. (C-12), which is accomplished by setting the first derivatives of Π_W and Π_T equal to zero. Before this is done, it is convenient to introduce the following approximating functions to describe the system at any interior point as a function of nodal values:

$$\phi = [N] \{\phi_N\} \ , \qquad (C\text{-}13)$$

$$\partial\phi/\partial x = [n,x] \{\phi N\} \ , \qquad (C\text{-}14)$$

$$C = [N'] \{C_N\}, \text{ and} \qquad (C\text{-}15)$$

$$\partial C/\partial x = [N',x]\ \{C_N\}\ , \tag{C-16}$$

where ϕ, C = potential and concentration respectively at any interior point; $\{\phi_N\}$ = ground-water potentials at nodal points in an element, a column matrix; $\{C_N\}$ = concentrations at nodal points in an element, a column matrix; and [N], [N'] = shape functions which relate nodal values of potential or concentration to values at any point in the element, a row matrix. This program allows each element to have a minimum of 4 and a maximum of 12 nodes. For example, if an element has six nodes the shape function has the following form:

$$\phi = [N]\ \{\phi_N\} = N_1\phi_1 + N_2\phi_2 + N_3\phi_3 + N_4\phi_4 + N_5\phi_5 + N_6\phi_6\ , \tag{C-17}$$

where $\phi_1 = \ldots \phi_6$ are nodal values of potential.

Derivation of the coefficients of the shape functions is not straightforward, but an understanding of how they are derived is not necessary for a general understanding of the finite element method. It is sufficient to know that if an element side has two nodes the shape function is linear, if the element side has three nodes the shape function is quadratic, and if the element side has four nodes the shape function is cubic. Therefore, the more nodes in an element, the higher the order of the polynominal used to approximate an interior point. This program uses isoparametric elements of the serendipity family. (See Pinder and Gray 1977 for more details.)

To solve the flow problem Eq. (C-11) is then minimized as follows:

$$\begin{aligned}\frac{\partial \Pi_W}{\partial\{\phi_N\}} = \quad & bK_{ij}[N,x_i]^T\ [N,x_j]\ \partial x_1 \partial x_2 \{\phi_N\} - W[N]\partial x_1 \partial x_2 \\ & - S\,\frac{\partial\{\phi_N\}}{\partial\{\partial t\}}\ ,\quad [N]^T\ [N]\ \partial x_1 \partial x_2 = 0\ .\end{aligned} \tag{C-18}$$

The integrations are performed using Gaussian quadrature (Zienkiewicz 1971). After the equation is integrated, it is convenient to combine terms and put the equation in the following form:

$$[S]\ \{\phi_N\} = [C]\ \frac{\partial\{\phi_N\}}{\partial t} - [R]\ , \tag{C-19}$$

where $[s] = bK_{ij}[N,x_i]^T\ [N,x_j]\ \partial x_1 \partial x_2$, which is often called the element structure matrix; $[c] = S\ [N]^T\ [N]\ \partial x_1 \partial x_2$, which is called the element capacitance matrix; and $[r] = W\ [N]\ \partial x_1 \partial x_2$, which is called the element recharge matrix.

An equation in the form of Eq. (C-19) is developed for each element in an aquifer. These equations are then added to give the following equation:

$$[S]\ \{\phi N\} = [C]\ \frac{\partial\{\phi N\}}{\partial t} - [R]\ , \tag{C-20}$$

where [S] = global structure matrix, [C] = global capacitance matrix, and [R] = global recharge matrix.

The column matrix will have one column for each node, and the square matrices will have a column and a row for each node. The square matrices will be symmetric and banded, and only half the matrix will be stored in the program.

The time derivative is evaluated by the Crank-Nicolson approximation:

$$([C] + \frac{\Delta t}{2} [S]) \{\phi_N\}^{i+1} = ([C] - [S] \frac{\Delta t}{2}) \{\phi_N\}^{i} + [R] \Delta t \ . \qquad (C\text{-}21)$$

The final matrix equation is solved by Gaussian elimination (Cook 1974).

To solve the mass or heat transport problem, Eq. (C-12) is minimized as follows:

$$\frac{\Pi_T}{\partial\{C_N\}} = [D_{ij} [N,x_i]^T [N,x_j] + g D'_{ij} [N,x_i]^T [N,x_j]] b dx_1 dx_2 \{C_N\}$$

$$- \quad g q_i [N,x_i]^T [N] b \partial x_1 \partial x_2 \{C_N\} - \rho \frac{\partial\{C_N\}}{\partial t} [N]^T [N] b \partial x_1 \partial x_2$$

$$+ H[N]^T [N] b dS \{C_N\} - H[N] C_\infty dS + q_i g [N]^T [N] b dS$$

$$\{C_N\} + q_i g C_\infty [N] b dS - R[N] b \partial x_1 \partial x_2 = 0 \ . \qquad (C\text{-}22)$$

This equation can be reduced to a form analogous to Eq. (C-19). As before, one equation is developed for each element, and these equations are summed to give an equation analogous in form to Eq. (C-20). The time derivative is again evaluated by the Crank-Nicolson approximation. The final matrix equation, which is assymmetric due to the presence of the terms $[N,x_i]^T[N]$ in Eq. (C-22), is solved by the Gauss-Doolittle method (Desai and Abel 1972).

CONSIDERATIONS IN DESIGNING A FINITE ELEMENT GRID

The basic step in using a finite element scheme is to subdivide the region of interest into an assemblage of smaller regions called elements. The process of discretizing is an exercise of engineering judgment in which one must choose the number, shape, size, and configuration of elements in such a way that the original continuum is simulated as closely as possible. A grid configuration that provides a greater number of nodes will generate a more accurate solution, but at the same time will lead to more computational effort. In general, the mesh should be refined in the region of steep gradients. The element shapes are quadrilaterals having straight boundaries, although curved boundaries are also possible.

All boundary conditions described for Eq. (C-1) and (C-4), except for constant flux boundaries, are handled explicitly by the program and are explained in the data-input section of the appendix. A constant flux boundary of zero flux is assumed by the finite element method if no other boundary condition is specified. A finite flux boundary is treated by assigning a line source or sink of the appropriate magnitude. Four sample problems will be presented to illustrate the discretization procedure.

In designing the finite element grid, 10 considerations should be observed.

1) All elements should be made nearly rectangular, since distorted element shapes decrease the solution accuracy.

2) Elements with more than four nodes should be used sparingly. Although an element with more than four nodes uses a higher order approximation function for interior points, replacing an element with six nodes by two elements with four nodes each usually provides as good an approximation with less computational effort. (This occurs because a lower order Gaussian quadrature scheme can be used for elements with only four nodes.) Elements with more than four nodes are best utilized for refining the finite element grid in critical areas (Figure C-2).

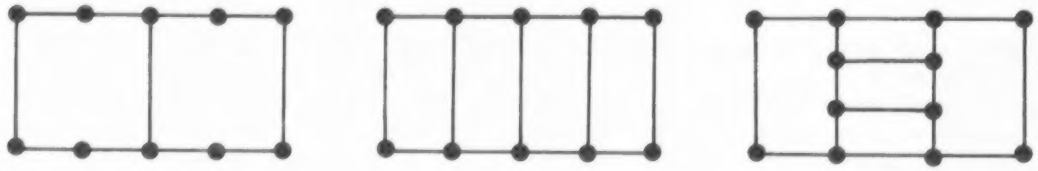


Figure C-2. Examples of refinement of a finite element grid. Left: bad use of multi-node sides; center: the preferred way of discretizing a region; right: good use of multi-node sides.

3) Nodes should be placed close together in areas where system parameters exhibit spatial changes, in areas where velocity is relatively rapid or the velocity distribution is complex, and in areas where a sharp temperature or concentration front is expected.

4) Exterior element boundaries across which a flux of mass or energy occurs must have only two nodes.

5) The velocity field near an exterior element boundary across which a flux of mass or energy occurs must not be complex. Preferably, all flow is normal to the boundary. This can be insured by setting up dummy elements along a boundary with a high hydraulic conductivity in a direction normal to the boundary. Likewise, the boundary should be oriented normal to one of the principle directions of the hydraulic conductivity tensor.

6) Boundaries within the area to be modeled should be located accurately. Distant boundaries can be located approximately with fewer nodes by expanding

the grid. Elements with more than four nodes are very useful for expanding the grid away from critical areas.

7) Constant concentration boundaries are generally unrealistic for the transport equation. Transport into the system can best be represented by multiplying the water flux across the boundary by the concentration exterior to the system.

8) The grid should be oriented to coincide with the principle directions of the hydraulic conductivity tensor.

9) The core requirements and computation time are proportional to the number of nodes representing the aquifer and to the number of times system parameters or time steps are changed.

10) If no boundary condition is specified along an exterior boundary, a no-flow boundary is assumed.

DATA DECK INSTRUCTIONS

The Data Groups

All data are read into the model with a free format, except for alphanumeric data. In free format, the data requested by one read statement may be put on one input card or as many input cards as the reader desires. Each bit of data, however, must be followed by either a comma or a blank. The data for each read statement must begin on a new card. Two read statements cannot read from the same card. The data and program output for four sample problems will be presented to illustrate the data deck preparation procedures. Several aspects of the data preparation that may be confusing are explained in more detail in the data notes following this section. The read statements that must be included on every program execution are underlined.

Group 1--

This group of data, which is read by both the main program and the data input subroutine, contains data required to dimension the model and data required to determine the type of problem.

Read Statement	Variable	Definition
1 and 2	TITLE	Any title that the user wishes to print on two lines at the start of output. The input must contain two cards.
3	LM	Number of elements
	LN	Number of nodes
	MBAND	Bandwidth (notes 1 and 2)

4	KTYPE	An integer that can have values in the range of one to five. 1--problem to be solved is a steady-state ground-water problem. 2--problem to be solved is a transient ground-water problem. 3--problem to be solved is a steady-state ground-water problem linked to a transient heat or mass transfer problem. 4--problem to be solved is a transient ground-water problem linked to a transient heat or mass transfer problem. 5--problem to be solved is a transient mass or heat transfer problem.
	KPRINT	An integer that can have values of 0 or 1. Set to 0 to suppress printing of input data; otherwise set to 1.
	LON	Set to 99 for calculating coefficient matrix stability for a transient problem (see note 3); otherwise set to zero.
	MQ	Set to 2 if one card is to be read in for each element giving the element node numbers. Set to 0 if the spatial structure for the problem is rectangular and all elements have only four nodes (data group I1).
	NLA	An integer specifying the method to be used for calculating velocities in a linked problem. Set to 1 if velocities are to be calculated at each of the Gauss points; this is generally the better method. Set to 2 if velocities are to be calculated only at the center of each element. Set to 0 if the problem is not linked problem.
	CFACT	Set as 1E13; this value is used to maintain the specified boundary conditions. Set higher if specified boundaries are not reproduced by model.
5 and 6	V(I)	Format statement for printing out nodal values. Two cards are required (note 4).
7 and 8	VV(I)	Format statement for printing out element values. Two cards are required.

Group II--

The data contained in this group of cards are required to specify the spatial structure of the nodal array. The type of data to be input is determined by the value previously specified for variable MQ. If the structure is rectangular with four nodes per element, the program automatically generates a grid and numbers the nodes and elements, and the data input is simple. If the structure is not rectangular, the location of each node must be input as must cards specifying the nodes defining each element (notes 5, 6, 7, and 8).

Read Statement	Variable	Definition
1	Q	Input the alphanumeric characters--up 11.
If MQ equals 2 only:		
one card for each node	J	node number
	XLOC	X location of the node
	YLOC	Y location of the node
one card for each element	J	element number
	NOD(K,1) NOD(K,2) NOD(K,3) NOD(K,4) NOD(K,5) . . NOD(K,1)	The element node numbers. The node numbers for each element must be listed counter-clockwise, starting at the corner nearest the origin. List the nodes in sequence; if a node is not a corner node, place an asterisk after it. Repeat the first node as the last value on each card.
If MQ equals 0 only:		
2	KSPACX	Number of nodes in the X direction.
	KSPACY	Number of nodes in the Y direction.
3	B(I)	KSPACX values, listing the X coordinates in order; smallest value should be listed first.
4	C(I)	KSPACY values, listing the Y coordinates in order; largest value should be listed first.
5	MLS	Optional (note 8).
6	D(I)	Optional (note 8).

Group III--

This group of data contains information on the initial conditions and on the specified boundary conditions. The data to be read in depends on the value specified for KTYPE.

Read Statement	Variable	Definition
1	Q	Input the alphanumeric characters-- Ill.
IF KTYPE equals 1:		
2	HEA(I)	Array used to specify constant potential boundaries. A zero is entered if a potential is not specified at the node; otherwise the specified potential value is entered. See note 9 for the format for entering these values.
IF KTYPE equals 2:		
2	ALPH	The length of each time step.
	ALPHM	Multiplication factor for each time step. Length of the next time step is equal to the length of the old time step multiplied by ALPHM. The program works most efficiently if ALPHM equals 1.
	KBOUN	Set to 1 if subroutine BOUND is to be called at each time step during execution of water-flow equation. Set to 2 if subroutine BVAL BVAL is to be called at each time step during execution of water-flow equation (note 13). Set to 99 when determining structure matrix stability for water-flow equation (note 3). Otherwise set to 0.
3	R1(I)	Array used to specify the initial potential at each node (use format explained in note 9).
4	HEA(I)	See above.
IF KTYPE equals 3:		
2	ALPHA	The length of time step for heat/mass equation.
	ALPHAM	Multiplication factor for the time step.
	KBOUND	An integer then can havve values 0, 1, 2, or 99. See KBOUN above, except KBOUND applies to the heat or mass flow equation.
	PCW	Heat capacity of water, or 1 in a mass transport problem with units of meters, grams, and ppm.
3	R(I)	An array used to specify the initial temperature or concentration at each node (note 9).

4	HEA(I)	See above. Specified potential values.
5	HEAD(I)	Array used to identify specified concentration boundaries. A zero is entered if temperature or concentration is not specified at the node; otherwise the specified value is entered (note 9).

IF KTYPE equals 4 or 5:

2	ALPH ALPHM BOUN	See description above.
3	R1(I)	See description above.
4	ALPHA ALPHAM BOUND PCW	See description above.
5	R(I)	See description above.
6	HEA(I)	See description above.
7	HEAD(I)	See description above.

Group IV--

This group of data contains information on the model parameters.

Read Statement	Variable	Definition
1	Q	Input the alphanumeric characters--Group IV.
2	NMATA	An integer, less than 51, specifying the number of different material types in the problem. A homogeneous area has only one material type.
3		This card contains multiplication factors for the parameters to be input later.
	FACTH	Factor for X direction hydraulic conductivity.
	FACTV	Factor for Y direction hydraulic conductivity.
	FACTWS	Factor for storage coefficient.
	FACTH1	Factor for X direction thermal conductivity.
	FACTV1	Factor for Y direction thermal conductivity.
	FACTHS	Factor for specific heat capacity of saturated medium.

	FACTDL	Factor for longitudinal dispersivity.
	FACTDT	Factor for transverse dispersivity.
4	MAT(J)	Type of material array. Integer values in the range of 1-50 are entered for each element, indicating the type of material in that element (format explained in note 9).
NMATA cards, one card for each material type.	J	Integer number in the range of 1-15 identifying the material type.
	WX	X-direction hydraulic conductivity
	WY	Y-direction hydraulic conductivity
	STO	Storage coefficient
	PX	X direction thermal conductivity, or molecular diffusion coefficient.
	PY	Y direction thermal conductivity or Y direction molecular diffusion coefficient.
	PCX	Heat capacity of the saturated medium, or in a mass transfer problem the porosity.
	DIFF(J,1)	Dispersivity coefficient, lateral.
	DIFF(J,2)	Dispersivity coefficient, transverse.

NOTE: Only values for J, WX, WY, and STO need be entered if KTYPE equals 1 or 2. If KTYPE equals 5, values for WX, WY, and STO must not be entered.

Group V--

This group of data contains information on sources and sinks. Line, areal, or point or sinks can be handled directly. Sources are positive, sinks are negative. If there are no or sinks, three cards must be entered, each with two zeros.

Read Statement	Variable	Definition
1	Q	Input the alphanumeric characters--Group V.
2	KGEN	Set to 1 if there is a uniform source or sink of water over at least one element in the problem; otherwise set to zero.
	KGENH	Set to 1 if there is a uniform source of energy or mass over one element in the problem; otherwise set to 0.
3	INFLOW(J)	Include this only if KGEN is not equal to 0. One value for each element specifying the rate of water generation per unit area per unit time in the element (note 9).
4	HEAT(J)	Include this only if KGENH is greater than 0. One value for each element specifying

		the rate of mass or heat generation per unit area per unit time in the element (note 9).
5	LWATER	Integer values specifying the number of point or sinks of water.
	LHEAT	The number of point sources or sinks of energy or mass.
6	NWATER(J)	Node number of a point source or sink of water.
	AWATER(J)	Rate of water pumpage per unit time. Enter sufficient cards to contain a node number and a rate for each water point source.
7	NHEAT(J)	Node number of point source of energy or mass.
	AHEAT(J)	Rate of energy or mass generation per unit time at the specified source. Enter sufficient cards to contain a node number and a rate for each point source.
8	LINEW	An integer specifying the number of line sources or sinks of water.
	LINEH	An integer specifying the number of line sources or sinks of heat.
9	NLINEW(I,1) NLINEW(I,2)	Element number of line source of water and boundary code (note 10). Enter sufficient cards to contain an element number and a code for each line source of water.
10	ALINEW(I)	Enter one value for each line source giving the rate of water recharge or discharge per unit length per unit time.
11	NLINEH(I,1) NLINEH(I,2)	Element number of line source of heat or mass and boundary code (note 8). Enter two values for each line source of heat or mass.
12	ALINEH(I)	Enter one value for each line source giving rate of heat input or output per unit length per unit time.

Group VI--

Data specifying if the problem is a cross-section problem or an areal problem.

Read Statement	Variable	Definition
1	Q	Input the alphanumeric characters--Group VI.
2	KAREAL	Set to 0 for cross-section problems; set to 1 for areal problems.
3	ER	Error criteria on ground-water potential for a steady-state problem. Solution is reached by an iterative procedure in which transmissivities are adjusted until the potential changes by less than the error criteria.
	ITER	Number of iterations permitted for a steady-state solution to be reached.
4	BOT(I)	Include only if KAREAL equals 1. Enter the bottom elevation of the aquifer at each node (note 9).
5	R1(I)	Initial potential values in the aquifer (use format explained in note 9). Do not input values if the initial values were input in data Group III.

Group VII--

This group of data contains information on the location of flow and convective flux boundaries. These types of boundaries are only used in the mass or heat transport equations. Skip cards 2-7 of this group if the problem is not a linked one. Flow and convective boundaries are discussed in data note 11.

Read Statement	Variable	Definition
1	Q	Input the alphanumeric characters--Group VII.
2	NCONV	An integer specifying the number of convective boundaries.
3	NCON(I,1)	Element number of the element containing this type of boundary.
	NCON(I,2)	This boundary code identifying the side across which the flux occurs.
	CONV(I)	The value of the convection (transfer) coefficient for a convective boundary.
		Include sufficient cards to contain three values for each boundary of this type.

4	TINF(I)	The temperature at a distance from the boundary (temperature at infinity). List one value for each boundary identified by card 3 of this group, and list them in the same order as on card 3.
5	LEL	An integer specifying the number of element sides across which a flux of water occurs.
6	NEL(I,1) NEL(I.2)	Element number and the boundary code. Include two values for each element boundary of this type.
7	AEL(I,2)	One value for each element boundary of this type specifying the temperature or concentration of the incoming fluid.

Group VIII--

This group of data contains information needed for calculating a mass balance. The data identifies the exterior boundaries on which potential, temperature, or concentration are specified (refer to data note 12).

Read Statement	Variable	Definition
1	Q	Input the alphanumeric characters-- Group VIII.
2	LFLUXw	An integer specifying the number of exterior element sides on which potential is specified.
	LFLUXH	An integer specifying the number of exterior element sides on which temperature or concentration is specified.
3		Include only if LFLUXW is greater than 0. Use sufficient cards to list two values for each element side.
	NFLUXw(I,1)	Element number of element with a specified potential on an exterior side.
	NFLUXw(I,2)	Side index number, identifying which of the four element sides the flux is across. An integer in the range of 1 to 4. See explanation notes.
4	NFLUXH(I,1) NFLUXH (I,2)	Include only if LFLUXH is greater than zero. Format is the same as above, except that here boundaries must be identified on which temperature or concentration is specified.

Group 1X--

This group of data contains information that controls the flow of the program and allows access to routines for changing boundary conditions at each time step, and it includes a routine for a moving boundary in a cross-section problem (see data note 14).

Read Statement	Variable	Definition
1	Q	Input the alphanumeric characters--Group 1X.
2	LA	Set to 0 unless ENTRY LAKE is to be called; if so set to 1.
	LD	Set to 0 unless ENTRY CHANG or ENTRY CHAN is to be called; if so set to 1.
	LE	Set to 0 unless water flows are to recompute at uneven intervals; if so, LE should equal the number of intervals at which water flows are to be recomputed (must be less than 100).
	LF	Set to 0 unless the problem is to be posed in radial coordinates; if so, set to 1.
	LG	Set to 0 unless the problem is to have a moving boundary; if so, set LG equal to the number of nodes which are to move.
	LH	Set to 0.
3	NTIME(I)	Include only if LE is greater than 0. LE values listing the time steps at which flows are to be recomputed; values must be listed in increasing order.
4		Include only if LF is equal to 1. Input 1,1,1,0,0,0 if problem is to be quasi-radial with aquifer thickness being equal to the x coordinate. Input -1,1,1,0,0,0 if problem is to be quasi-radial with aquifer thickness being equal to the y coordinate.
5		Input values needed by ENTRY LAKE.
6		Input values needed by subroutine ADJUST (note 14).
If KTYPE equals 1:		
7	MW	Set to 1 if potentials are to be written on file 14; otherwise set to 2.

If KTYPE equals 2:

7	MA	Total number of time units for the simulation.
	MB	Number of time steps between printing of head values.
	MW	Number of time steps between writing heads on file 14.
	MC	Number of time steps between printing of mass balance.
	MD	Number of time steps between printing of flows. (MA, MB, MW, MC, and MD must not equal zero.)

If KTYPE equals 3, 4, or 5:

7	MO	Total number of time units for the simulation.
	MP	Number of time steps between printing nodal values.
	MV	Number of time steps between printing nodal values on file 14.
	MQ	Number of time steps between printing of mass balance; if set to a negative number, mass balances will not be computed which results in a considerable savings in CPU time.
	MR	Number of time steps between recomputing water flows.
	MS	Set to 1 if initial hydraulic conductivity values are to be adjusted for changing temperature in the aquifer; otherwise set to 0 (note 15).
	MT	Set to 1 if using dispersivities greater than zero; otherwise set to 0.
	MU	Set to 1 if water flows are not to be printed when flows are recomputed; otherwise set to 0. (MO, MP, MV, MQ, and MR must not equal zero.)
	ME	Set to 1 if the program is to stop after water flows are computed; otherwise set to 0.

Data Explanation Notes

1) Program size--

The storage required for the program is approximately

$$2000 + LN(11 + MBAND \times 4) + LM \times 25,$$

where LN is the number of nodes, LM is the number of elements, and MBAND is the bandwidth.

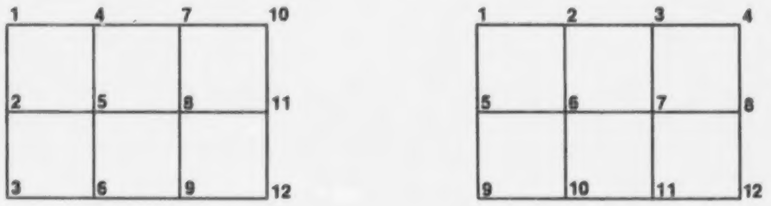
The second statement in the main program is used to change the amount of storage allocated for the program. The parameter variable N specifies the maximum number of nodes, the parameter variable L specifies the maximum number of elements, and the parameter variable M specifies the maximum bandwidth. On a UNIVAC 1110, if the program is compiled in FORTRAN V, the combination of 297 nodes, 260 elements, and a bandwidth of 13 is the maximum permissible. This arrangement uses 64K of memory. If the program is compiled in ASCII FORTRAN on a UNIVAC 1110, the program size could be quadrupled.

If the program is being used only to solve potential problems, the value of the parameter variable M in statement of 1 of the main program need only be equal to one-half the bandwidth plus one. For a linked problem M must be equal to or less than the bandwidth.

Also, if LWATER, LHEAT, NWATER, NHEAT, LINEW, LINEH (data Group V), NCONV, LEL (data Group VII), LFLUXW, or LFLUXH (data Group VIII) exceed 50, statement 1 of the main program must be changed. The parameter variable Z must be changed from Z = 50 to Z = the maximum value of the above-listed variables.

2) Bandwidth--

The bandwidth depends upon the largest difference between any two nodes in a single element and is equal to the maximum difference between any two nodes in an element in a structure plus one (Figure C-3).



(a) bandwidth equals five

(b) bandwidth equals six

Figure C-3. Examples of numbering of nodes in a structure.

The nodes in a structure should be numbered so as to make the bandwidth as small as possible.

3) Calculation of Stability for a Transient Problem--

The EIGEN subroutine computes the maximum (critical) time step that can be used in a transient simulation to insure that the solution computed by the Crank-Nicolson method will be nonoscillatory. The routine is based on a modified Eigen value extraction technique developed by Myers (1978). The scheme slightly underestimates the maximum (critical) time step. The equation used to estimate the critical time step is

$$\Delta t_c = \frac{1}{2} \min_{i=1}^{LN} \left(\frac{C_{i,i}}{S_{i,i} + \sum\limits^{LN} | S_{i,j} |} \right) \quad \begin{matrix} j=1 \\ j\neq 1 \end{matrix}$$

where Δt_c = safe estimate of the critical time step, $S_{i,j}$ = the jth entry in the ith row of the structure matrix, and $C_{i,i}$ = the diagonal term in the ith row of the capacitance matrix.

The critical time step is a function of system parameters, the x and y spacing, and boundary conditions. The routine prints out the maximum time step for each node in the structure and prints out the critical time step.

4) Standard FORTRAN formats--

Standard FORTRAN formats are used for printing node and elements values. Values are printed consecutively starting with the value for node or element 1. The format input statements allow the user to specify how the values are to be printed out. This option can be very helpful if an irregular grid is used.

Typical formats for printing out node and element values for a grid 5 nodes by 5 nodes would be: (1X,5(5G12.6/)) for the node values; and (1X,4(4G12.6/)) for the element values. (The word FORMAT is not used; the '(' goes in column one of the data card.) The node values would then be printed out on five lines, five values on each line.

5) Orientation of the Finite Element Grid--

The finite element grid must be orientated correctly, since flows and boundary conditions will be treated incorrectly if the grid is orientated incorrectly. The program requires a standard cartesian orientation (Figure C-4).

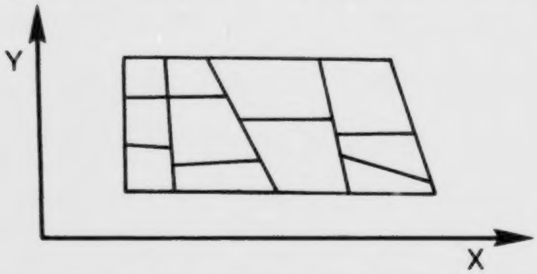
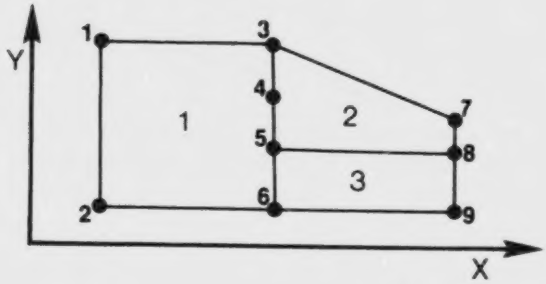


Figure C-4. Cartesian orientation of finite element grid.

Values in the x direction must increase from left to the right and values in the y direction must increase from the bottom to the top.

6) Element Node Numbering--

Typical elements are shown in Figure C-5 to illustrate the manner in which element node numbers must be input to the program.



The input data:

element #	the node numbers						
1	2	6	5*	4*	3	1	2
2	5	8	7	3	4*	5	
3	6	9	8	5	6		

Figure C-5. Typical elements of a finite element grid and the correct method of numbering the elements.

7) The Rectangular Grid Generator--

The rectangular grid generator generates a grid in which the nodes and elements are numbered in the form shown in Figure C-6.

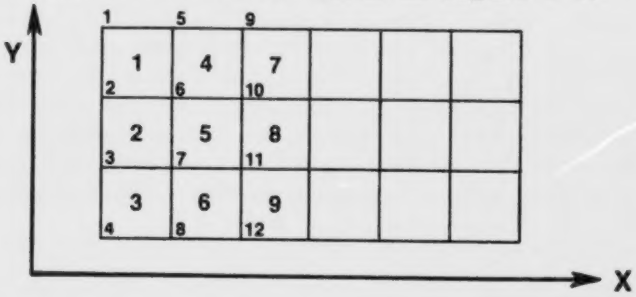


Figure C-6. Numbering of nodes and elements in a rectangular grid.

The program assigns to node 1 the first x and y coordinates read in. For proper orientation the list of x-coordinate values read in must be arranged so the smallest value is read in first, and the list of y coordinates must be arranged so that the largest value is read in first.

8) Selection of Variants of Rectangular Grid--

The program can create the two variants of a rectangular grid shown in Figure C-7.

To select the options shown in Figure C-7:

1) set KSPACX = -KSPACX.

2) set MLS = 0 for option (a); set MLS = 1 for option (b).

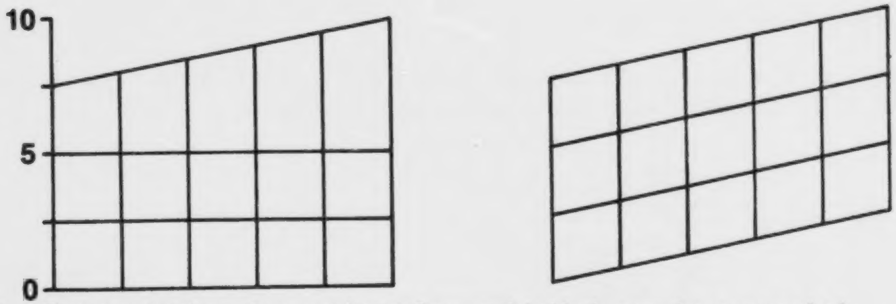


Figure C-7. Two variants of a rectangular grid that can be generated. Left: grid with sloping upper surface; right: grid with all rows sloping.

3) Input KSPACX values for the D array. Input one value for each set of nodes in the x direction. These values specify how much the y coordinates of the nodes are to be adjusted from the values specified by C(I). For the examples illustrated above the values in the D array would be: 0, 0.5, 1.0, 1.5, 2.0, 2.5. These values need not increase or decrease monotonically.

9) Format for Reading in Array Data--

Several options are provided to the user to simplify the often arduous task of initializing array value. The user must initially specify one value to be assigned to every position in the array. The user then has the options of (1) leaving the array as is, (2) reinitializing the entire array by inputting one value for each position in the array, or (3) selectively changing the initialization. The last option is accomplished by specifying the array position that is to be reinitialized and the new value. An array position will correspond to a node or element number. This format may be used when initializing R, R1, HEA, HEAD, MAT, and BOT.

The format to be used is:

Read Statement	Variable	Definition
1	QI	Value to be assigned initially to each position in the array.
	IZ	Three options: a. set to -1 if one value is to be input for each node or element. b. set to 0 if all values in the array equal QI. c. set to a positive integer specifying the number of array values to be different from QI.
	FACT	Factor by which each of the array values is to be multiplied.

If IZ = -1:

2 (sufficient cards to contain one value for each node or element)	Enter the values to be assigned to each node or element. Values are to be entered consecutively beginning with node or element 1. After one value is entered for each node or element, enter 999 which is a sentinel character.
If IZ - 0:	No cards are needed.
If IZ > 0:	
2 (sufficient cards to contain IZ x 2 values)	Enter a node or element number and the value to be assigned to this node or element. Enter two values for each array position that is to be reinitialized.

10) Boundary Codes--

When specifying line fluxes and boundary fluxes, the side of an element across which the flux is occurring must be designated. The sides of an element are coded as shown in Figure C-8.

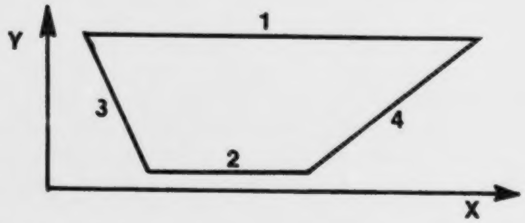


Figure C-8. Numbering of sides of an element.

11) Flow Boundaries--

Data Group VII allows the user to specify several types of boundary fluxes. The type of boundary fluxes permitted across an element side are:

convective, where transfer is equal to $AH(T_i - T\infty)$;

flow out, where transfer is equal to $V_i T_i A$;

flow in, where transfer is equal to $V_i T\infty A$;

where A = the length of the element side multiplied by the aquifer thickness; H = a transfer coefficient; for heat transfer the units are, H/L^2tT; T_i = temperature or concentration on the boundary; $T\infty$ = temperature or concentration beyond the boundary; and V_i = water velocity normal to the boundary.

If the boundary is to have more than one type of flux, for example, a flow and a convective flux, it should be treated for purposes of data input as though there were two separate boundaries.

12) Mass Balance--

The mass balance routine computes an approximate mass balance. Several types of boundary fluxes are computed only approximately by the mass balance routine, and often the mass balance errors calculated by this routine will be much higher than they really are. The program generally has mass balance errors of less than 1% for both the water-flow and the transport equations. The user must identify the specified head and temperature or concentration boundary conditions. The program identifies the other boundary conditions and sources and sinks.

13) Boundary Conditions That Vary With Time--

Because the program could handle a variety of boundary conditions that might change at each time step, a program to cover all possibilities could not be written. The user must write additional routines to change boundary values at each time step.

Several entry points in subroutine BOUNDA are provided for this. Starting addresses for all arrays in which a user is likely to change values at each time step are passed to this subroutine as well as the time step counter KS. The entry points available in this subroutine are:

(a) ENTRY LAKE

The routine is called if LA = 1 (data Group IX). This routine is called immediately after the data input routine and is called only once for a given program execution. The routine can be used to read in information to be used in altering boundary conditions during execution of the program.

(b) ENTRY BOUND

Called during solution of the water-flow equation by subroutine LOAD if KBOUN (data Group III) equals 1, or called during solution of the transport equation by subroutine LOAD if KBOUND equals 1.

(c) ENTRY BVAL

Called during solution of the water-flow equation by subroutine LOAD if KBOUN equals 2, or called during solution of the transport equation by subroutine LOAD if KBOUND equals 2.

(d) ENTRY CHANG

Called during solution of the transport equation by subroutine LOAD if LD = 1 (data Group IX).

(e) ENTRY CHAN

Called during solution of the water-flow equation by subroutine LOAD if LD = 1. The program logic is shown on the program flow chart, which shows the position of the calls to the entry points in BOUNDA by subroutine LOAD.

14) Documentation for Moving Boundary Routine--

For cross-section problems the surface boundary can be programmed to move in response to changing recharge rates. Only boundaries with boundary code 1 may move (note 10). The moving boundary routine is called by setting the 5th value (LG) on card 1, Group IX, equal to the number of nodes that will move. The following cards are then added in the appropriate position in data group IX.

Read Statement	Variable	Definition
1	NSTEP	Number of time periods to be stimulated.
	NPRINT	Number of time periods between printing of potentials.
	NPR	Number of time periods between printing flows and mass balance.
	ERROR	Change allowed in node location before structure matrix is recomputed.
	FACTOR	Factor to be multiplied by recharge rates.
2		Four values for each moving boundary:
	NMOV(J,1)	Element number on left of node*,
	NMOV(J,2)	Element number on right of node*,
	BMOV(J,3)	Storage coefficient,
	BMOV(J,6)	Initial recharge rate.

*Note: If several nodes are to be constrained to rise and fall at the same rate, the left and right element number for each node of this type must be preceded by a minus sign (-).

After the last card of Group IX one card is added for each pumping period.

Read Statement	Variable	Definition
	BKS	Identifier for this time period.
	TIME	Length of this time period.
	FACT	Factor that relates initial recharge rates to recharge rates at this time step. The current recharge rates are calculated by multiplying FACT by initial recharge rate.

15) Hydraulic Conductivity as a function of Temperature--

The program in subroutine PE adjusts hydraulic conductivities for changing temperature distributions. The following relations are used:

$$\mu = (1.917 - 0.05635T + 0.0071T^2) \times 10^{-3}, \text{ and}$$

$$K_i = K_{15^0} \times 1.21/\mu ,$$

where μ = the kinnematic viscosity in centimeter-gram-second units, and T = the temperature in degrees centigrade.

The relationship is only valid for temperatures between 0° and 50°C. The relation also assumes that the initial hydraulic conductivities are specified at a temperature of 15°C. The relationship could easily be reprogrammed to cover a different temperature range or to relate hydraulic conductivities to concentration.

DERIVATION OF A HEAT TRANSPORT EQUATION

The processes that control the transport of heat in an aquifer are in many ways analogous to the processes that control the transport of mass in an aquifer. The partial differential equation describing the transport of mass is well known, and its derivation is clearly explained in Konikow and Grove (1977) and is rigorously derived in Bear (1972). Rather than derive the heat transport equation from first principles, which would in many respects repeat Konikow and Grove (1977), we show how the mass transport equation can be modified to apply to the transport of heat.

These assumptions are made in this derivation:

1) The aquifer is incompressible and chemically inert with respect to the fluid;

2) Fluid density is constant;

3) Hydraulic head is the only driving force; coupled processes, Onsanger relationships, and density driven convections are not considered;

4) Fluid flow is laminar; and

5) Divergence of velocity equals zero.

In addition to these five assumptions the generalized mass transport equation

$$\frac{\partial}{\partial x_i}\left(D_{ij}\frac{\partial C}{\partial x_j}\right) + \frac{\partial}{\partial x_i}\left(D'_{ij}\frac{\partial C}{\partial x_j}\right) - q_i\frac{\partial C}{\partial x_i} - R - n\frac{\partial C}{\partial t} = 0 , \qquad \text{(C-23)}$$

as derived by Bear (1972) and Konikow and Grove (1977), assumes the following about the processes that transport mass in an aquifer: (1) Velocity driven convection, molecular diffusion, and mechanical dispersion are the only transfer processes; and (2) no transfer of mass occurs within the solid phase.

In an aquifer the basic modes of heat transfer are (Lagarde 1965, quoted in Bear 1972, p. 640) heat transfer through the solid phase by conduction, heat transfer through the fluid phase by conduction, heat transfer through the fluid phase by convection, and heat transfer by dispersion.

Each of the terms in Eq. (C-23) is discussed sequentially to demonstrate how a similar form for each term can be used to describe the transport of heat in an aquifer.

Molecular Diffusion

Heat, unlike mass, is readily transmitted through the solid phase of most porous media if a temperature gradient exists. In the derivation of the first term of Eq. (C-23), it was assumed that transport does not occur in the solid phase. The coefficient of molecular diffusion in the mass transport equation is defined on the basis of tortuosity (Bear 1972), a concept that is applicable if transfer only takes place through the fluid phase.

The transfer of heat by molecular diffusion (conduction) through the solid phase and the fluid phase can be treated by assuming a simple parallel conduction model in which conduction through the fluid phase occurs separately but simultaneously with no interchange of heat between the two media. The real situation is more complex since heat is interchanged continuously between the two phases. Experimental data (Houpert 1965, quoted in Bear 1972, p. 646) suggest that the simple model is sufficient, and the assumption of simultaneousness is valid if the time period is greater than a few minutes.

Therefore, heat conduction can be described by

$$\frac{n\partial}{\partial x_i} (kf_{ij} \frac{\partial C}{\partial x_j}) + (1-n) \frac{\partial}{\partial x_i} (ks_{ij} \frac{\partial C}{\partial x_j}) , \qquad (C-24)$$

where kf_{ij} = coefficient of thermal molecular diffusion for the fluid phase, ks_{ij} = coefficient of thermal molecular diffusion for the solid phase, and n = the porosity.

Dispersive Transfer of Heat

The form of the second term in Eq. (C-23) was derived from experimental data for mass transport in porous media. Green (1963) concludes on the basis of experimental data that in the range of Darcian flow the influence of the passage of heat through the solid phase will be insignificant and that the dispersive transfer of mass and heat will be identical. (Dispersive transfer is a term used to describe convective transfer that occurs due to velocity fluctuations in the fluid.)

Internal Generation of Heat

Heat will be generated by a moving fluid because of energy dissipation by viscous stresses. The rate of dissipation, E, can be estimated from

$$E = \rho q g \frac{\partial \phi}{\partial x} ,$$

where g = the gravitational constant and ρ = fluid density.

A fluid flowing at a rate of 1 m/day, with a gradient of 1 m/m, will dissipate energy at the rate of 10^{-13} cal/m^3 per day. Therefore, heat dissipation is considered to be negligible.

The Absorption of Heat by the Solid Phase

The absorption of heat by the solid phase can be treated analogously to the adsorption of mass by the solid phase when the process is characterized by a linear adsorption isotherm. The last term of E. (C-23) can be modified (Bear 1972, Pickens and Lennox 1977) to

$$n \frac{\partial C}{\partial t} + (1-n)\ A \frac{\partial C}{\partial t} \tag{C-25}$$

to treat the adsorption of mass by the solid phase, where A = the adsorption distribution coefficient that describes the ratio of solute on the solid phase to solute in the fluid phase.

If A is defined to be the ratio of heat in the solid phase to heat in the fluid phase, Eq. (C-25) can be used to describe the absorption of heat by the solid phase in a porous medium.

The heat transport equation can then be obtained by substituting the terms developed above into Eq. (C-23):

$$\frac{\partial}{\partial x_i} [(nkf_{ij} \frac{\partial C}{\partial x_j}) + (1-n)\ ks_{ij} \frac{\partial C}{\partial x_j})] + (D'_{ij} \frac{\partial}{\partial x_i}) - q_i \frac{\partial C}{\partial x_i} - R - n \frac{\partial C}{\partial t} - (1-n)\ A \frac{\partial C}{\partial t} = 0\ . \tag{C-26}$$

By convention the concentration of heat is generally expressed as a temperature, where concentration equals temperature multiplied by the heat capacity. The heat capacities of a fluid and solid phase are generally not the same. Each term in Eq. (C-26) refers to heat transfer in either the solid or the fluid phase; no term refers to a combined heat transfer. Making the appropriate substitutions, Eq. (C-26) can be written as:

$$\frac{\partial}{\partial x_i} [(n\rho C_w kf_{ij} \frac{\partial T}{\partial x_j}) + (1-n)\ \rho C_{sol} ks_{ij} \frac{\partial T}{\partial x_j})] + \frac{\partial}{\ }\ \rho C_w\ (D_{ij} \frac{\partial T}{\partial x_j}) - \rho C_w q_i \frac{\partial T}{\partial x_i} - R - \rho C_w\ \frac{\partial T}{n_{\partial t}} - \rho C_w\ (1-n)\ A \frac{\partial T}{\partial t} = 0\ , \tag{C-27}$$

where ρC_w = heat capacity of the fluid, H/L^3°C; and ρC_{sol} = heat capacity of the solid phase, H/L^3°C. The equation can be simplified by introducing the following conventions.

1) $K_{ij} = (1-n)\ \rho C_{sol} ks_{ij} + n\rho C_w kf_{ij}$,

where K_{ij} are the components of the thermal conductivity tensor for the saturated media. It is much simpler to determine experimentally the combined coefficient than the individual ones, which are a function not only of phase composition but also of phase geometry.

2) The absorption distribution coefficient $A = \rho C_{sol}/\rho C_w$. This relationship holds by definition.

3) The heat capacity of a whole is equal to the sum of the heat capacities of its parts: $\rho C_s = n\rho C_w + (1-n)\rho C_{sol}$.

Equation (C-27) then reduces to the following form when the substitutions are made which is in the same form as Eq. (C-23):

$$\frac{\partial}{\partial x_i}\left(K^T_{ij}\frac{\partial T}{\partial x_j}\right) + \frac{\partial}{\partial x_i}\rho C_w \left(D_{ij}\frac{\partial T}{\partial x_j}\right) - \rho C_w q_i \frac{\partial T}{\partial x_i} - R - \rho C_s \frac{\partial T}{\partial t} = 0 , \qquad (C\text{-}28)$$

SAMPLE PROBLEMS

Linear Heat Transport

This example problem demonstrates the use of all linear elements and the use of mixed higher order elements to solve a one-dimensional convective heat transport problem. The problem analyzed and the finite element grids used to discretize the problem are shown in Figure C-9. The parameters used in the problem are listed in Table C-2. The input data and the program-generated output for both sample simulations are listed in Figures C-10, C-11, C-12, and C-13. The simulated temperatures are nearly identical for both the grid with mixed elements and the one with all linear elements (Table C-3). Simulated temperatures differ by less than 0.5°C.

In analogy to Ogata and Banks (1961), an analytical solution to the linear convective heat transport problem is:

$$T = T_o/Z \left[\mathrm{erfc}\left(\frac{x-q't}{Z(K't/\rho C_s)^{1/2}}\right) + \exp\left(\frac{xq\rho C_w}{K'}\right)\mathrm{erfc}\left(\frac{x+q't}{Z(K't/\rho C_s)^{1/2}}\right)\right] , \qquad (C\text{-}29)$$

where $q' = q\rho C_w/\rho C_s$, and $K' = K^t + q\alpha_T\,\rho C_w$, in which the following boundary conditions are used:

$$T(x,0) = 0,\ x > 0;$$
$$T(0,t) = T_o,\ t \geq 0;\ \text{and}$$
$$T(\infty,t) = 0,\ t \geq 0.$$

The analytical solutions at t = 25, 50, 75 days are shown with the simulated temperatures in Figure C-14. The analytical solution at t = 50 days is compared to the simulated temperatures at t = 50 days in Table C-3. In addition, the simulated temperatures for the all linear element grid with nodes 21 and 22 specified at 15°C at t = 50 days are also presented in Table

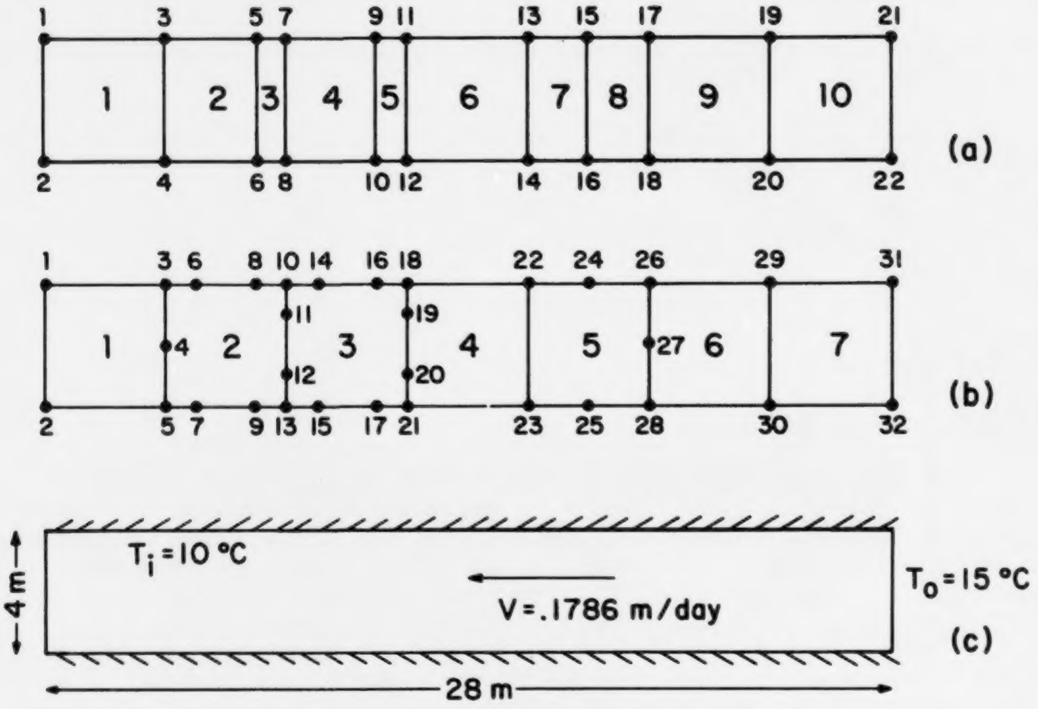

1 3 5 7 9 11 13 15 17 19 21
1 2 3 4 5 6 7 8 9 10
2 4 6 8 10 12 14 16 18 20 22
(a)
1 3 6 8 10 14 16 18 22 24 26 29 31
11 19
4 27
12 20
1 2 3 4 5 6 7
2 5 7 9 13 15 17 21 23 25 28 30 32
(b)
$T_i = 10$ °C
V = .1786 m/day
$T_o = 15$ °C
4 m
28 m
(c)

Figure C-9. Finite element grids used to discretize a linear heat transport problem. The problem is depicted in (c). The grid with all linear elements is shown in (a), and the grid with mixed higher order elements is shown in (b). Both node numbers and element numbers are shown in (a) and (b).

```
Convective test problem rectangular matrix
Units used are meters, days, celsius, calories

10 22 4
3 0 0 0 1 1E13
(1x,2G12.6)

(1x,7G12.6)

GROUP II
11 2
0 4 7 8 11 12 16 18 20 24 28
4 0
GROUP III
.5 1 0 1E6
10 0 1
0 4 1
1 100 2 100 21 105 22 105
0 0 1
GROUP IV
1
1 1 1 1 1 1 1 1
1 0 1
1 1 1 0 55000 55000 700000 .1 .1
GROUP V
0 0
0 0
0 0
GROUP VI
0
GROUP VII
1
1 3 0
10
1
10 4 0
15
GROUP VIII
2 0
1 3 10 4
GROUP IX
0 0 0 0 0 0 0 0
75 50 150 50 150 0 1 0
```

Figure C-10. The input data used to model one-dimensional heat transport with all linear elements.

TABLE C-2. PARAMETERS, INITIAL CONDITIONS, AND BOUNDARY CONDITIONS USED FOR THE HEAT-FLOW EQUATION IN THE LINEAR HEAT TRANSPORT PROBLEM[a]

	Parameters	Initial conditions	Boundary conditions
V	0.1786 m/day	T_i = 10°C at all nodes	Dirichlet (convective flow) boundaries at $x = 0$ and $x = 28$
K^t	0.55×10^5 cal/m day °C		
ρC_w	0.1×10^7 cal/m^2 °C		Temperature of incoming fluid: 15°C
ρC_s	0.7×10^6 cal/m^2 °C		
time step	0.5 day		No flow at all other boundaries

---

[a] The water-flow equation was solved for head gradient of 0.1786 m/m with hydraisic conductivity and porosity set to unity to obtain the specified velocity.

```
               CONVECTIVE TEST PROBLEM RECTANGULAR MATRIX
               UNITS USED ARE METERS, DAYS, CELSIUS, CALORIES

                                                        04//07/78
                                                        17:01:09
#OF NODES 22 # OF ELEMENTS 10 BANDWIDTH  4

VELOCITIES ARE BEING CALCULATED AT CURRENT TIME STEP     1

THE X-SPACING IS

   0.0        4.0000     7.0000     8.0000     11.0000     12.0000     16.0000     18.0000
 20.0000     24.0000    28.0000

THE Y-SPACING IS
   4.0000     0.0

EQUATION  2 TIME STEP  .500000          HEAT OR MASS CAPACITY COEFFICIENT  .100000+07

PARAMETER FACTORS--IN ORDER   1.00000  1.00000  1.00000  1.00000  1.00000  1.00000  1.00000  1.00000

  MATERIAL  HOR PERM  VER PERM    STORAGE   HOR THERM COND   VERT THERM COND SPECIFIC DISPERSION COEFS
                                                                              HEAT
                1 1.000   1.000      .0000       .5500+05            .5500+05   .7000+06 .1000+00 .1000+00

THE VALUES IN THE TYPE OF MATERIAL MATRIX ARE
  1.00000    1.00000    1.00000    1.00000    1.00000    1.00000    1.00000
  1.00000    1.00000    1.00000

INFORMATION FOR MASS OR HEAT TRANSFER ACROSS A SPECIFIED HEAD BOUNDARY
ELEMENT NUMBER--BOUNDARY CODE--TEMPERATURE OR CONCENTRATION OF INCOMING FLUID
    10   4    15.000
CONVECTIVE OUT AND CONDUCTIVE BOUNDARY INFORMATION        ELEM #--BOUNDARY CODE--TRANSFER COEFFICIENT
     1 3  .000000
```

Figure C-11. Program output for one-dimensional heat transport problem with linear elements.(continued)

```
WATER FLOW BOUNDARIES--ELEM # AND BOUNDARY CODE
  1  3        10   4

POTENTIAL DISTRIBUTION AT TIME STEP     .000000
 100.0000    100.0000
  100.714     100.714
  101.250     101.250
  101.429     101.429
  101.964     101.964
  102.143     102.143
  102.857     102.857
  103.214     103.214
  103.571     103.571
  104.286     104.286
  105.000     105.000
                               CUMULATIVE MASS BLANCE          RATES FOR THIS STEP
B. FLUX RECHARGE                     .714288                         .714288
B. FLUX DISCHARGE                    .714281                         .714281
CHANGE IN STORAGE                    .000000                         .000000
QUANTITY PUMPED                      .000000                         .000000
DIFFERENCE                           .715256-05                      .715256-05

TIME STEP   .000000
FLOWS IN THE X DIRECTION
  .714281   .714284  .714285  .714286  .714287  .714286  .714287
  .714288   .714288  .714288

FLOWS IN THE Y DIRECTION
  .000000  .715256-06  .119209-06  .357628-06  .000000  .476837-06  .000000
  .000000 -.476837-06  .000000

THE DISPERSION ROUTINE IS BEING USED
```

Figure C-11. (continued)

```
TEMPERATURE DISTRIBUTION AT TIME STEP      25.0000
   10.0001     10.0002
   9.99935     9.99949
   10.0014     10.0016
   10.0074     10.0074
   9.99971     9.99992
   9.96143     9.96156
   9.90413     9.90422
   10.3286     10.3286
   11.4248     11.4248
   14.1568     14.1569
   15.3485     15.3486

                                 CUMULATIVE MASS BLANCE      RATES FOR THIS TIME STEP
   CONDUCTIVE TRANSFER                 .000000                     .000000
   CONVECTIVE TRANSFER--OUT            .178571+09                  .357147+07
   CONVECTIVE TRANSFER--IN             .267858+09                  .535716+07
   B. FLUX RECHARGE                    .000000                     .000000
   B. FLUX DISCHARGE                   .000000                     .000000
   CHANGE IN STORAGE                   .892868+08                  .178568+07
   QUANTITY PUMPED                     .000000                     .000000
   DIFFERENCE                          429.500                     8.37500

TEMPERATURE DISTRIBUTION AT TIME STEP      50.0000
   10.0037     10.0039
   9.97586     9.97601
   9.94439     9.94460
   10.0029     10.0029
   10.4781     10.4783
   10.9284     10.9285
   12.8394     12.8395
   14.0114     14.0115
   14.5750     14.5750
   15.1468     15.1468
   14.9093     14.9093
```

Figure C-11. (continued)

```
                            CUMULATIVE MASS BLANCE   RATES FOR THIS TIME STEP
CONDUCTIVE TRANSFER               .000000                 .000000
CONVECTIVE TRANSFER-OUT           .357165+09              .357283+07
CONVECTIVE TRANSFER-IN            .535716+09              .535716+07
B. FLUX RECHARGE                  .000000                 .000000
B. FLUX DISCHARGE                 .000000                 .000000
CHANGE IN STORAGE                 .178550+09              .178431+07
QUANTITY PUMPED                   .000000                 .000000
DIFFERENCE                        896.781                 14.2500

TEMPERATURE DISTRIBUTION AT TIME STEP       75.0000
  10.0302   10.0304
  10.5080   10.5082
  11.6597   11.6599
  12.1306   12.1306
  13.4834   13.4837
  13.8607   13.8608
  14.9332   14.9333
  14.9278   14.9278
  15.0309   15.0309
  15.0045   15.0045
  14.9912   14.9913

                            CUMULATIVE MASS BLANCE   RATES FOR THIS TIME STEP
CONDUCTIVE TRANSFER               .000000                 .000000
CONVECTIVE TRANSFER-OUT           .535367+09              .358047+07
CONVECTIVE TRANSFER-IN            .803574+09              .535716+07
B. FLUX RECHARGE                  .000000                 .000000
B. FLUX DISCHARGE                 .000000                 .000000
CHANGE IN STORAGE                 .268205+09              .177668+07
QUANTITY PUMPED                   .000000                 .000000
DIFFERENCE                        1372.56                 7.93750
```

Figure C-11. Program output for one-dimensional heat transport problem with linear elements.

```
CONVECTIVE TEST PROGRAM
UNITS USED ARE METERS, DAYS, CELCIUS, CALORIES

7  32  12
3  0  0  2  1  1E13
(1x,2G12.6/3G12.6/2G12.6/2G12.6/4G12.6/2G12.6/2G12.6/4G12.6/2G12.6/2G12.6/
3G12.6/2G12.6/2G12.6)
(1x,7G12.6)

GROUP II

1  0  4
2  0  0
3  4  4
4  4  2
5  4  0
6  5  4
7  5  0
8  7  4
9  7  0
10  8  4
11  8  3
12  8  1
13  8  0
14  9  4
15  9  0
16  11  4
17  11  0
18  12  4
19  12  3
20  12  1
21  12  0
22  16  4
23  16  0
24  18  4
25  18  0
26  20  4
27  20  2
28  20  0
29  24  4
30  24  0
31  28  4
32  28  0
1  2  5  4*  3  1  2
2  5  7*  9*  13  12*  11*  10  8*  6*  3  4*  5
3  13  15*  17*  21  20*  19*  18  16*  14*  10  11*  12*  13
4  21  23  22  18  19*  20*  21
5  23  25*  28  27*  26  24*  22  23
6  28  30  29  26  27*  28
7  30  32  31  29  30
```

(continued)

Figure C-12. The data deck used to model one-dimensional heat transport with mixed elements.

```
GROUP III

.5  1  0  1E6
10  0  1
0  4  1
1  100  2  100  31  105  32  105
0  0  1

GROUP IV

1
1  1  1  1  1  1  1  1
1  0  1
1  1  1  0  55000  55000  700000  .1  .1

GROUP V

0  0
0  0
0  0

GROUP VI

0

GROUP VII

1
1  3  0
10
1
7  4  0
15


GROUP VIII

2  0
1  3  7  4

GROUP IX

0  0  0  0  0  0  0  0
75  50  150  50  150  0  1  0
```

Figure C-12. The data deck used to model one-dimensional heat transport with mixed elements.

```
                  CONVECTIVE TEST PROGRAM
                  UNITS USED ARE METERS, DAYS, CELCIUS, CALORIES

                                                                  04/07/76
                                                                  17:00:46

# OR NODES    32  # OF ELEMENTS    7 BANDWIDTH    12

VELOCITIES ARE BEING CALCULATED AT CURRENT TIME STEP       1

EQUATION 2 TIME STEP   .500000           HEAT OR MASS CAPACITY COEFFICIENT  .100000+07

PARAMETER FACTORS--IN ORDER   1.00000    1.00000   1.00000   1.00000   1.00000   1.00000   1.00000   1.00000

   MATERIAL  HOR PERM    VER PERM     STORAGE      HOR THERM COND      VERT THERM COND   SPECIFIC    DISPERSION
                                                                                         HEAT        COEFS
             1   1.000      1.000       .0000       .5500+05            .5500+05          .7000+06   .1000+00 .1000+00

THE VALUES IN THE TYPE OF MATERIAL MATRIX ARE
   1.00000   1.00000   1.00000   1.00000   1.00000   1.00000 1.00000

INFORMATION FOR MASS OR HEAT TRANSFER ACROSS A SPECIFIED HEAD BOUNDARY
ELEMENT  UNSER--BOUNDARY CODE--TEMPERATURE OR CONCENTRATION OF INCOMING FLUID
     7   4   15.000
CONVECTIVE OUT AND CONDUCTIVE BOUNDARY INFORMATION   ELEM #--BOUNDARY CODE--TRANSFER COEFFICIENT
     1   3       .000000

WATER FLOW BOUNDARIES--ELEM # AND BOUNDARY CODE
  1  3         7  4

POTENTIAL DISTRIBUTION AT TIME STEP    .000000
  100.0000    100.0000
  100.714     100.714   100.714
  100.893     100.893
  101.253     101.253
  101.429     101.429   101.429   101.429
  101.657     101.657
  101.964     101.964
  102.143     102.143   102.143   102.143
  102.857     102.857
```

(continued)

Figure C-13. Program output for the one-dimensional heat transport problem with mixed elements.

```
   103.214      103.214
   103.571      103.571    103.571
   104.236      104.236
   103.000      105.000
                                CUMULATIVE MASS BLANCE        RATES FOR THIS TIME STEP
 B. FLUX RECHARGE                      .714289                      .714289
 B. FLUX DISCHARGE                     .714202                      .714262
 CHANGE IN STORAGE                     .000000                      .000000
 QUANTITY PUMPED                       .000000                      .000000
 DIFFERENCE                            .267029-04                   .267029-04

 TIME STEP      .000000
 FLOWS IN THE X DIRECTION
    .714262      1.42996      1.74388      .915181      .714290    .714292      .714289

 FLOWS IN THE Y DIRECTION
   -.476837-56   .558137-05   .540157-05   -.548363-05  .00-000  -.953674-06   .000000

 THE DISPERSION ROUTINE IS BEING USED

 TEMPERATURE DISTRIBUTION AT TIME STEP     25.0000
    10.0005     10.0006
    10.0001     9.99989    10.0002
    9.99925     9.99977
    10.0025     10.0026
    10.007?     10.0058    10.0058    10.0076
    10.0073     10.0079
    9.99686     9.9551
    9.90961     9.97127    9.97589    9.96879
    9.?985?     9.89?2?
    10.3231     10.3?3?
    11.4719     11.4715    11.4717
    14.1362     14.1365
    13.3683     15.3502
                             CUMULATIVE MASS BLANCE      RATES FOR THIS TIME STEP
 CONDUCTIVE TRANSFER               .000000                         .000000
 CONVECTIVE TRANSFER-OUT           .178569+09                      .357150+07
 CONVECTIVE TRANSFER-IN            .267858+09                      .535716+07
 B. FLUX  RECHARGE                 .000000                         .000000
 B. FLUX DISCHARGE                 .000000                         .000000
 CHANGE IN STORAGE                 .589193+08                      .180833+07
 QUANTITY PUMPED                   .000000                         .000000
 DIFFERENCE                        370063.                         -22663.4
```

Figure C-13. (continued)

```
TEMPERATURE DISTRIBUTION AT TIME STEP     50.0000
   10.0049    10.0050
   9.96511    9.93?23   9.98017
   9.96235    9.96254
   9.96261    9.95263
   10.0013    9.95-12   9.99911 10.0018
   1?.1391    1?.1396
   1?.54??    1?.54?4
   1?.??3     1?.3997   10.8995 10.8954
   12.??73    12.9877
   13.9537    13.9556
   14.5316    14.5311   14.5313
   15.1661    15.1663
   14.8985    14.5984

                              CUMULATIVE MASS BLANCE       RATES FOR THIS TIME STEP
CONDUCTIVE TRANSFER              .000000                       .000000
CONVECTIVE TRANSFER-OUT          .357160+09                    .357313+07
CONVECTIVE TRANSFER-IN           .535716+09                    .535716+07
B. FLUX RECHARGE                 .000000                       .000000
B. FLUX DISCHARGE                .000000                       .000000
CHANGE IN STORAGE                .17257+09                     .179040+07
QUANTITY PUMPED                  .000000                       .000000
DIFFERENCE                       .330007+07                   -6372.50

TEMPERATURE DISTRIBUTION AT TIME STEP    75.0000
  10.8259      10.0261
10.5035       1?.5926       10.5035
10.6??9       1?.9895
11.6724       11.6724
12.1127       12.1154       12.1154      12.1135
12.5?53       12.5?66
13.4747       13.4742
13.?692       13.5693       13.8603      13.86?5
14.9299       14.93?4
14.9297       14.9296
15.0453       15.044?       15.0451
15.??1?       15.??22
14.9917       14.9917
```

Figure C-13. (continued)

```
                                       CUMULATIVE MASS BLANCE      RATES FOR THIS TIME STEP
CONDUCTIVE TRANSFER                           .000000                     .000000
CONVECTIVE TRANSFER-OUT                       .635375+09                  .357859+07
CONVECTIVE TRANSFER-IN                        .000000                     .000000
B. FLUX RECHARGE                              .000000                     .000000
B. FLUX DISCHARGE                             .000000                     .000000
CHANGE IN STORAGE                             .567842+09                  .183896+07
QUANNTITY PUMPED                              .000000                     .000000
DIFFERENCE                                    356718.                     -60682.3
EXGT
```

Figure C-13. Program output for the one-dimensional heat transport problem with mixed elements.

TABLE C-3. ANALYTICAL SOLUTION AT t= 50 DAYS FOR THE PROBLEM POSED IN FIGURE C-9 AND FINITE ELEMENT NUMERICAL SOLUTIONS AT t = 50 DAYS FOR TWO GRID CONFIGURATIONS AND TWO TYPES OF BOUNDARY CONDITIONS

	Analytical	Linear elements (a)	Mixed elements	Linear elements with nodes 21 & 22 specified at 15°C
x = 0	15.0[a]	14.901	14.899	15.0
x = 4	14.982	15.147	15.166	15.081
x = 8	14.649	14.575	14.531	14.484
x = 10	14.017	14.011	13.945	13.981
x = 12	12.962	12.840	12.867	13.036
x = 16	10.787	10.929	10.900	10.724
x = 20	10.062	10.003	9.999	10.095
x = 24	10.002	9.976	9.976	9.763

[a] All values listed are temperatures in degrees centigrade. x = 0 is at the right in Figure C-10.

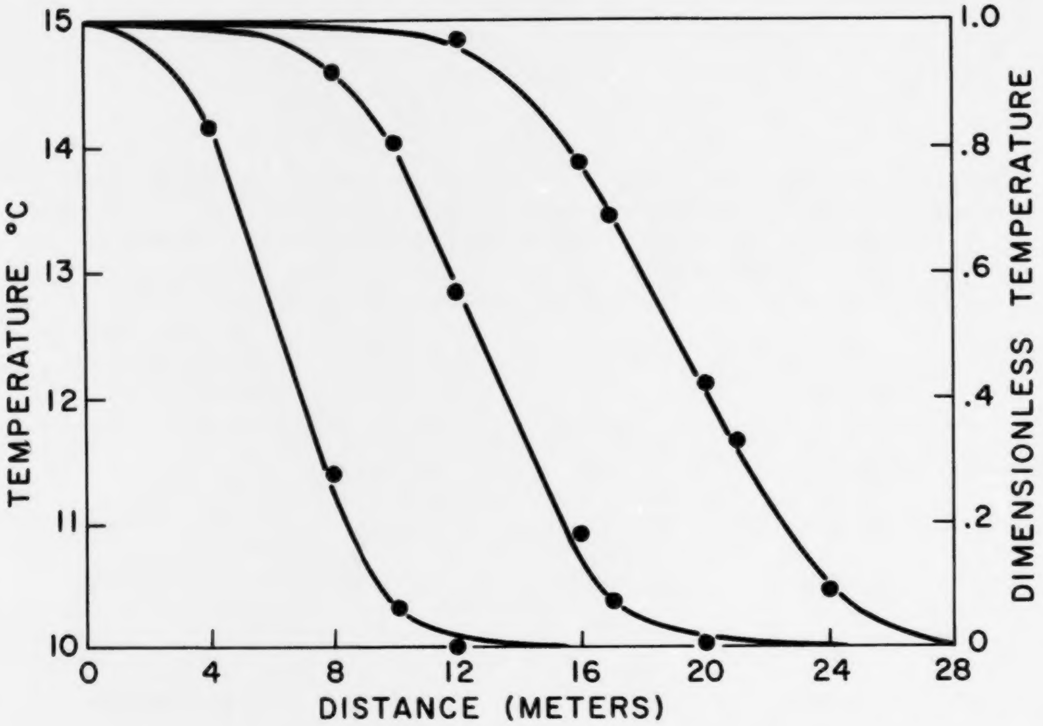


Figure C-14. Analytical solutions (solid lines) and numerical solutions (dots) for the linear heat transport problem for t = 25, 50 and 75 days.

C-3. Specification of the boundary temperature, rather than use of a convective boundary, constrains the solution from oscillating as much near the origin and is the exact boundary condition used by Ogata and Banks (1961).

The Mohawk River

This problem demonstrates the use of the program to solve an areal problem to investigate the effect of stream infiltration on ground-water temperatures in an alluvial aquifer along the Mohawk River near Schenectady, N.Y. (Figure C-15).

The flood plain of the Mohawk River in the vicinity of Lock and Dam 8, about 2 miles west of the city of Schenectady, is underlain by more than 100 ft of unconsolidated deposits. From the surface downward the deposits consist of 30 ft of flood-plain alluvium, 20-100 ft of sandy gravel and sand, and, immediately above the bedrock, a layer of glacial till 25-50 ft thick (Figure C-16). The sandy gravel and sand deposit is tapped by the well fields of the city of Schenectady and town of Rotterdam. Winslow (1962) monitored temperatures in this aquifer and concluded that most of the water pumped from the well fields originated as infiltration from the Mohawk River. This model was programmed to determine if the temperature patterns observed by Winslow could be simulated.

The grid used to model the aquifer is shown in Figure C-17. The boundary conditons and parameters used in the simulation are listed in Table C-4. All water flow was assumed to occur within the principal aquifer, and the principal aquifer was assumed to be thermally insulated from the overlying flood-plain deposits and the underlying glacial tills. (This assumption greatly simplified the data input, but it could be relaxed if a better approximation is desired.)

The temperature of the river water was changed at each time step. River temperatures for a 1-yr period were read in at the beginning of program execution by a call to ENTRY LAKE and were then altered at each time step by a call to ENTRY BOUND. ENTRY CHANG was called at each time step to compute heat flow out of the system with the pumped water. Heat flow at the wells for each time step was set equal to the temperature at the wells multiplied by the heat capacity and the rate of pumpage.

The data and the program output for a 10-day simulation, with pumpage rates only 10% of actual rates, are listed in Figure C-18 and Figure C-19. If actual pumpage rates are used, a time step of 0.1 day must be used to insure solution stability, unless a corrector is applied near the wells to force stability.

TABLE C-4. PARAMETERS, INITIAL CONDITIONS, AND BOUNDARY CONDITIONS USED FOR THE SIMULATION OF TEMPERATURES IN THE MOHAWK RIVER ALLUVIAL AQUIFER

Water flow	
Boundary conditions	
Mohawk River	Specified head of 100 ft[a]
All other boundaries	No flow
Parameters (principle aquifer)	
$K_{11} = K_{22}$	100,000 gal/day ft^2
Wells--discharge rates	
Node 70	2,000,000 gal/day
Nodes 130, 131, 145, 146	4,500,000 gal/day at each node
Heat flow	
Initial conditions	$T_i = 10^oC$
Boundary conditions	
River	Convective flux in
All other boundaries	No flow
Parameters	
$K^t_{11} = K^t_{22}$	1.3×10^4 cal/day ft°C
ρC_s	2.1×10^4 cal/ft^3 °C
α_L	10 ft
α_T	1 ft
Time step	1 day

[a] Units of feet, gallons, calories, and days were used in this simulation.

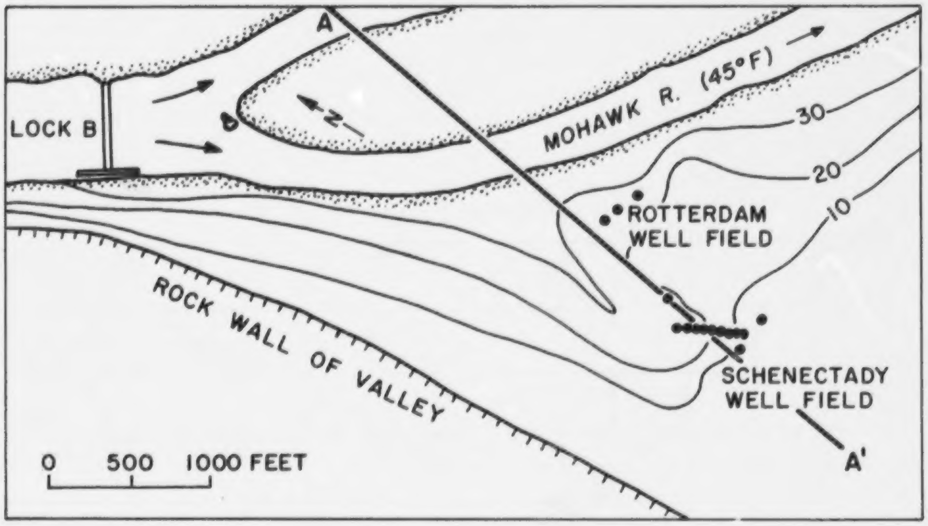

A
LOCK B
B
N
MOHAWK R. (45°F)
30
20
10
ROTTERDAM WELL FIELD
ROCK WALL OF VALLEY
SCHENECTADY WELL FIELD
A'
0 500 1000 FEET

Figure C-15. Areal view of the Mohawk River Valley showing location of the Schenectady and Rotterdam well fields (Winslow 1962).

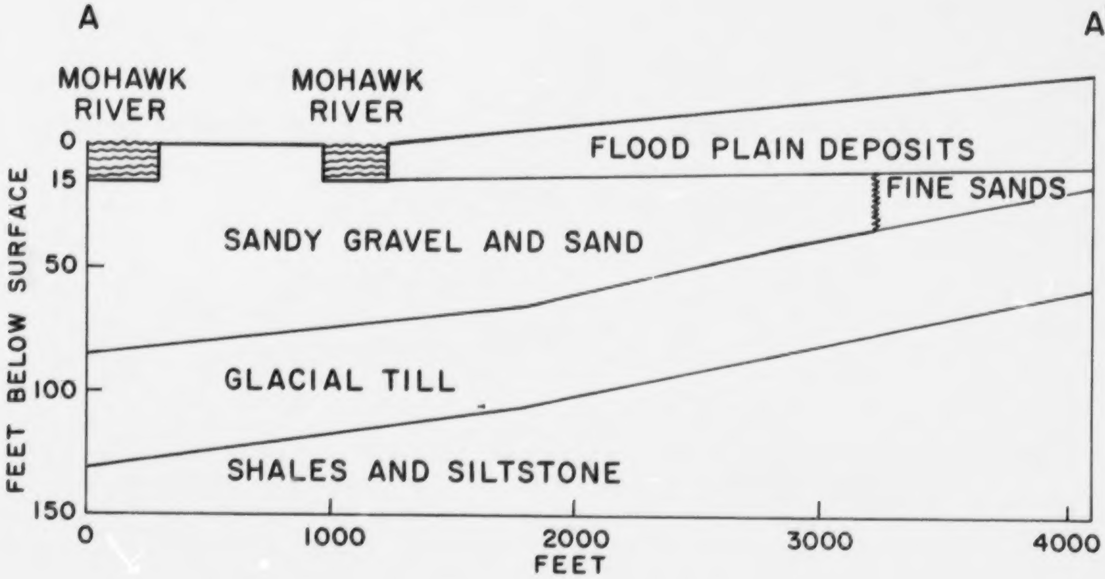

A
A'
MOHAWK RIVER
MOHAWK RIVER
FLOOD PLAIN DEPOSITS
FINE SANDS
SANDY GRAVEL AND SAND
GLACIAL TILL
SHALES AND SILTSTONE
FEET BELOW SURFACE
0
15
50
100
150
0
1000
2000
3000
4000
FEET

Figure C-16. Cross-sectional view of the Mohawk River alluvial aquifer along section A-A' of Figure C-15.

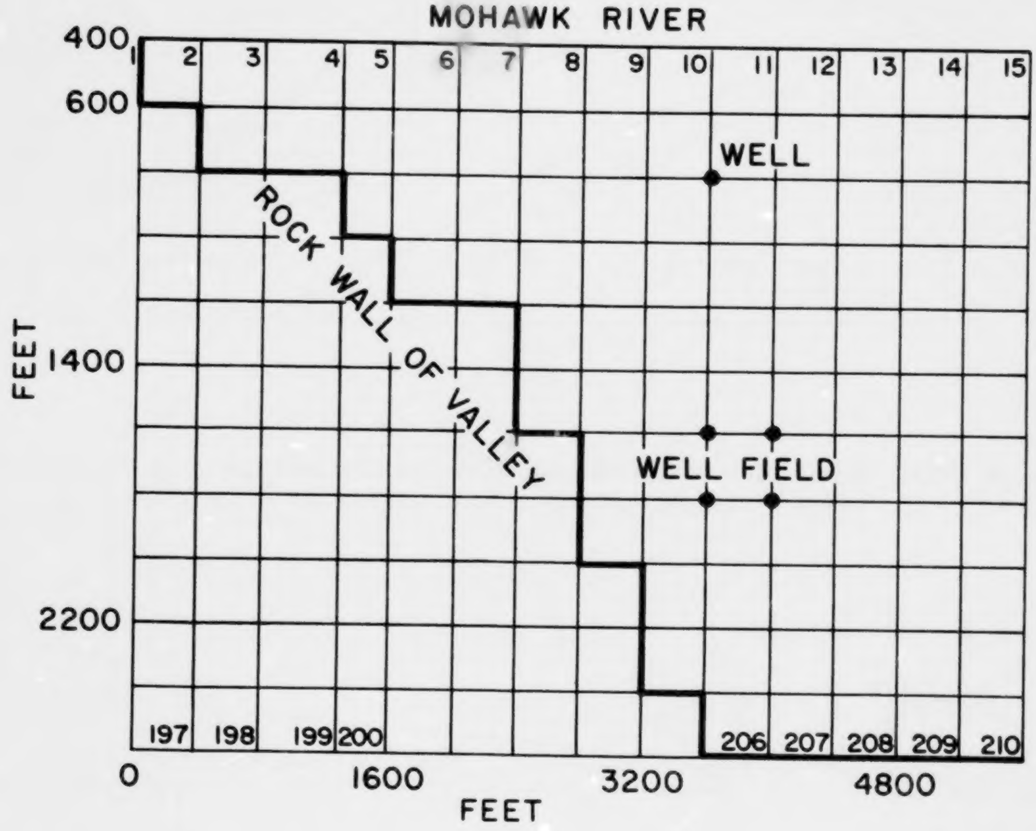

MOHAWK RIVER
400
600
1400
2200
FEET
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
WELL
ROCK WALL OF VALLEY
WELL FIELD
197
198
199
200
206
207
208
209
210
0
1600
3200
4800
FEET

Figure C-17. Grid used to discretize the Mohawk River problem. (The node numbers in the first and last rows are listed.)

```
  MOHAWK RIVER PROBLEM
  UNITS ARE GALLONS DAYS FEET
182   210 17
3  0  0  0  1  1E16
(1x,15F6.2)

(1x,14F6.2)

GROUP II
14  15
400  450  500  600  800  1000  1200  1400  1600  1800  2000   2200  2400  2600
5600  5200  4800  4480  4000  3600  3200  2800  2400  2000  1600  1200  800  400  0
GROUP III
1  1  1  3785.68
10 0 1
0  -1  1
 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100
                              0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
999
 0 0 1
GROUP IV
4
1 1 1 1 1 1 1 1 1
0 -1 1
1 1 1 1 1 1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2 2 2 2 2 2
2 2 2 2 2 2 2 2 2 2 2 2 2 2
4 2 2 2 2 2 2 2 2 2 2 2 2 2
4 4 4 2 2 2 2 2 2 2 2 2 2 2
4 4 4 4 2 2 2 2 2 2 2 2 2 2
4 4 4 4 4 4 2 2 2 2 2 2 2 2
4 4 4 4 4 4 2 2 2 2 2 2 2 2
4 4 4 4 4 4 4 2 2 2 2 2 2 2
4 4 4 4 4 4 4 2 2 2 2 2 2 2
4 4 4 4 4 4 4 4 3 3 3 3 3 3
4 4 4 4 4 4 4 4 3 3 3 3 3 3
4 4 4 4 4 4 4 4 3 3 3 3 3 3 999
1 100000 1000000 0 13000 13000 21000 10 1
2 100000 1000000 0 13000 13000 21000 10 1
3 500 500 0 13000 13000 21000 10 1
4 0.001 0.001 0.001 0.001 0.001 0.001 0.001 0.001
GROUP VI
0 0
5 5
130 -4.5E5 131 -4.5E5 145 -4.5E5 146 -4.5E5 70 -2E5
130 1 131 1 145 1 146 1 70 1
```

(continued)

Figure C-18. The data deck used to model heat flow in the Mohawk River alluvial aquifer.

```
0 0
GROUP VI  1
1
1 3
0 -1 1
1 1 1 1 1 10 20 30 40 50 50 50 50 50 50
1 1 1 1 1 10 20 30 40 50 50 50 50 50 50
1 1 1 1 1 10 20 30 40 50 50 50 50 50 50
1 1 1 1 1 10 20 30 40 50 50 50 50 50 50
1 1 1 1 1 10 20 30 40 50 50 50 50 50 50
99 99 99 20 20 25 35 45 50 55 55 55 55 55 55
99 99 99 99 40 45 50 55 60 60 60 60 60 60 60
99 99 99 99 99 99 65 65 65 60 60 60 60 60 60
99 99 99 99 99  99 70 70 70 70 70 70 70 70 70
75 75 75 75 75 75 75 75 75 75 75 75 75 75 75
80 80 80 80 80 80 80 80 80 80 80 80 80 80 80
80 80 80 80 80 80 80 80 80 80 80 80 80 80 80
80 80 80 80 80 80 80 80 80 80 80 80 80 80 80
80 80 80 80 80 80 80 80 80 80 80 80 80 80 80 999
100 0 1
GROUP VII
14
1 3 3000 2 3 3000 3 3 3000 4 3 3000 5 3 3000 6 3 3000 7 3 3000
8 3 3000 9 3 3000 10 3 3000 11 3 3000 12 3 3000 13 3 3000 14 3 3000
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
14
1 3 2 3 3 3 4 3 5 3 6 3 7 3 8 3 9 3 10 3 11 3 12 3 13 3 14 3
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
GROUP VIII
14 0
1 3 2 3 3 3 4 3 5 3 6 3 7 3 8 3 9 3 10 3 11 3 12 3 13 3 14 3
GROUP IX
1 1 0 0 0 0
0 220 220 1 220 220 1 220 220 1 1 1
277 860
     1.0000          29.5000          41.0000
     2.0000          28.5000          35.9000
     3.0000          27.5000          30.8000
     4.0000          27.4600          30.5000
     5.0000          27.4100          30.2000
     6.0000          27.3700          29.9000
     7.0000          27.3300          29.6000
     8.0000          27.2900          29.3000
     9.0000          27.2400          29.0000
    10.0000          27.2000          28.7000
10 10 10 10 10 1 1 1
```

Figure C-18. The data deck used to model heat flow in the Mohawk River alluvial aquifier.

```
                    MOHAWK RIVER PROBLEM
                    UNITS ARE GALLONS DAYS FEET
                                                        04/07/78
                                                        17:01:49
# OF NODES 210  # OF ELEMENTS 182  BANDWIDTH 17

VELOCITIES ARE BEING CALCULATED AT CURRENT TIME STEP 1

THE X-SPACING IS
  400.0000     450.0000     500.0000     600.0000     800.0000    1000.0000    1200.0000    1400.0000
 1600.0000    1800.0000    2000.0000    2200.0000    2400.0000    2600.0000

THE Y-SPACING IS:
 5600.0000    5200.0000    4800.0000    4480.0000    4000.0000    3600.0000    3200.0000    2800.0000
 2400.0000    2000.0000    1600.0000    1200.0000     800.0000     400.0000      0.0

EQUATION 2 TIME STEP 1.00000               HEAT OR MASS CAPACITY COEFFICIENT  3785.68

PARAMETER FACTORS- IN ORDER     1.00000   1.00000   1.00000   1.00000   1.00000   1.00000   1.00000   1.00000

    MATERIAL    HOR PERM    VER PERM    STORAGE    HOR THERM COND    VERT THERM COND    SPECIFIC DISPERSION COEFS
                                                                                         HEAT
                1  .1000+06  .1000+06   .0000        .1300+05          .1300+05          .2100+05  10.00   1.000

                2  .1000+06  .1000+06   .0000        .1300+05          .1300+05          .2100+05  10.00   1.000

                3  500.0     500.0      .0000        .1300+05          .1300+05          .2100+05  10.00   1.000

                4  .1000-02  .1000-02   .1000-02     .1000-02          .1000-02          .1000-02  .1000-02.1000-02
THE VALUES IN THE TYPE OF MATERIAL ARE
  1.00  1.00  1.00  1.00  1.00  1.00  1.00  1.00  1.00  1.00  1.00  1.00  1.00  1.00
  2.00  2.00  2.00  2.00  2.00  2.00  2.00  2.00  2.00  2.00  2.00  2.00  2.00  2.00
  2.00  2.00  2.00  2.00  2.00  2.00  2.00  2.00  2.00  2.00  2.00  2.00  2.00  2.00
  4.00  2.00  2.00  2.00  2.00  2.00  2.00  2.00  2.00  2.00  2.00  2.00  2.00  2.00
  4.00  4.00  4.00  2.00  2.00  2.00  2.00  2.00  2.00  2.00  2.00  2.00  2.00  2.00
  4.00  4.00  4.00  4.00  2.00  2.00  2.00  2.00  2.00  2.00  2.00  2.00  2.00  2.00
  4.00  4.00  4.00  4.00  4.00  4.00  2.00  2.00  2.00  2.00  2.00  2.00  2.00  2.00
  4.00  4.00  4.00  4.00  4.00  4.00  2.00  2.00  2.00  2.00  2.00  2.00  2.00  2.00
  4.00  4.00  4.00  4.00  4.00  4.00  4.00  2.00  2.00  2.00  2.00  2.00  2.00  2.00
  4.00  4.00  4.00  4.00  4.00  4.00  4.00  2.00  2.00  2.00  2.00  2.00  2.00  2.00
  4.00  4.00  4.00  4.00  4.00  4.00  4.00  4.00  3.00  3.00  3.00  3.00  3.00  3.00
  4.00  4.00  4.00  4.00  4.00  4.00  4.00  4.00  3.00  3.00  3.00  3.00  3.00  3.00
  4.00  4.00  4.00  4.00  4.00  4.00  4.00  4.00  3.00  3.00  3.00  3.00  3.00  3.00
```

Figure C-19. Program output for Mohawk River problem. (continued)

```
POINT SOURCES OF WATER--NODE # AND AMOUNT
   130   -450000.      131   -450000.      145   -450000.      146   -450000.      70   -200000.

POINT SOURCES OF HEAT
   130   1.00000       131   1.00000       145   1.00000       146   1.00000       70   1.00000

ELEVATION OF THE AQUIFER BOTTOM
   1.00  1.00   1.00   1.00   1.00  10.00  20.00  30.00  40.00  50.00  50.00  50.00  50.00  50.00  50.00
   1.00  1.00   1.00   1.00   1.00  10.00  20.00  30.00  40.00  50.00  50.00  50.00  50.00  50.00  50.00
   1.00  1.00   1.00   1.00   1.00  10.00  20.00  30.00  40.00  50.00  50.00  50.00  50.00  50.00  50.00
   1.00  1.00   1.00   1.00   1.00  10.00  20.00  30.00  40.00  50.00  50.00  50.00  50.00  50.00  50.00
   1.00  1.00   1.00   1.00   1.00  10.00  20.00  30.00  40.00  50.00  50.00  50.00  50.00  50.00  50.00
  99.00 99.00  99.00  20.00  20.00  25.00  35.00  45.00  50.00  55.00  55.00  55.00  55.00  55.00  55.00
  99.00 99.00  99.00  99.00  40.00  45.00  50.00  55.00  60.00  60.00  60.00  60.00  60.00  60.00  60.00
  99.00 99.00  99.00  99.00  99.00  99.00  65.00  65.00  65.00  65.00  60.00  60.00  60.00  60.00  60.00
  99.00 99.00  99.00  99.00  99.00  99.00  70.00  70.00  70.00  70.00  70.00  70.00  70.00  70.00  70.00
  75.00 75.00  75.00  75.00  75.00  75.00  75.00  75.00  75.00  75.00  75.00  75.00  75.00  75.00  75.00
  80.00 80.00  80.00  80.00  80.00  80.00  80.00  80.00  80.00  80.00  80.00  80.00  80.00  80.00  80.00
  80.00 80.00  80.00  80.00  80.00  80.00  80.00  80.00  80.00  80.00  80.00  80.00  80.00  80.00  80.00
  80.00 80.00  80.00  80.00  80.00  80.00  80.00  80.00  80.00  80.00  80.00  80.00  80.00  80.00  80.00
  80.00 80.00  80.00  80.00  80.00  80.00  80.00  80.00  80.00  80.00  80.00  80.00  80.00  80.00  80.00

INFORMATION FOR MASS OR HEAT TRANSFER ACROSS A SPECIFIED HEAD BOUNDARY
ELEMENT NUMBER--BOUNDARY CODE--TEMPERATURE OR CONCENTRATION OF INCOMING FLUID
  1  3   .00000      2  3     .00000      3  3    .00000      4  3    .00000      5  3    .00000
  6  3   .00000      7  3     .00000      8  3    .00000      9  3    .00000     10  3    .00000
 11  3   .00000     12  3     .00000     13  3    .00000     14  3    .00000

CONVENTIVE OUT AND CONDUCTIVE BOUNDARY INFORMATION        ELEM #--BOUNDARY CODE--TRANSFER COEFFICIENT
  1  3   3000.00     2  3     3000.00     3  3    3000.00     4  3    3000.00     5  3    3000.00
  6  3   3000.00     7  3     3000.00     8  3    3000.00     9  3    3000.00    10  3    3000.00
 11  3   3000.00    12  3     3000.00    13  3    3000.00    14  3    3000.00

WATER FLOW BOUNDARIES--ELEM # AND BOUNDARY CODE
  1  3      2  3     3  3    4  3     5  3     6  3      7  3     8  3     9  3     10  3
 10  3     11  3    12  3   13  3    14  3
```

Figure C-19. (continued)

```
ITERATIONS NEEDED FOR CONVERGENCE     1

POTENTIAL DISTRIBUTION AT TIME STEP    .000000
  100.00  100.00  100.00  100.00  100.00  100.00  100.00  100.00  100.00  100.00  100.00  100.00  100.00  100.00  100.00
  100.00  100.00  100.00  100.00  100.00  100.00  100.00  100.00   99.99   99.99   99.99   99.99   99.99  100.00  100.00
  100.00  100.00  100.00  100.00  100.00  100.00  100.00   99.99   99.99   99.98   99.98   99.99   99.99   99.99   99.99
  100.00  100.00  100.00  100.00  100.00  100.00   99.99   99.98   99.97   99.96   99.97   99.97   99.98   99.98   99.98
  100.00  100.00  100.00  100.00  100.00   99.99   99.98   99.97   99.95   99.92   99.93   99.94   99.96   99.97   99.97
  100.00  100.00  100.00  100.00   99.99   99.99   99.97   99.95   99.92   99.89   99.89   99.91   99.93   99.95   99.95
   99.97   99.97   99.98   99.99   99.99   99.99   99.96   99.93   99.89   99.84   99.84   99.88   99.91   99.93   99.94
   99.94   99.94   99.94   99.96   99.98   99.97   99.94   99.91   99.85   99.78   99.78   99.84   99.89   99.92   99.93
   99.91   99.91   99.91   99.91   99.91   99.91   99.93   99.88   99.82   99.68   99.68   99.82   99.88   99.91   99.92
   99.90   99.90   99.90   99.90   99.89   99.89   99.89   99.84   99.79   99.63   99.63   99.80   99.87   99.90   99.91
   99.90   99.90   99.90   99.90   99.89   99.88   99.87   99.83   99.77   99.65   99.65   99.78   99.86   99.90   99.91
   99.90   99.90   99.90   99.90   99.89   99.86   99.85   99.81   99.72   99.69   99.70   99.77   99.84   99.88   99.90
   99.90   99.90   99.90   99.89   99.89   99.87   99.84   99.79   99.70   99.70   99.72   99.77   99.83   99.87   99.89
   99.90   99.90   99.90   99.89   99.89   99.87   99.84   99.79   99.70   99.70   99.72   99.77   99.83   99.87   99.88

                                  CUMULATIVE MASS BLANCE                RATES FOR THIS TIME STEP
B. FLUX RECHARGE                    .200989+07                            .200989+07
B. FLUX DISCHARGE                   .000000                               .000000
CHANGE IN STORAGE                   .000000                               .000000
QUANTITY PUMPED                    -.200000+07                           -.200000+07
DIFFERENCE                          9886.42                               9886.42


THE DISPERSION ROUTINE IS BEING USED
NODE # AND TEMP AT INF.  1  15.04   2  15.04   3  15.04   4  15.04  5  15.04  6  15.04  7  15.04  6  15.04
   9  15.04  10  15.04  11  15.04  12  15.04  13  15.04  14  15.04
NODE # AND TEMP AT INF.  1  15.52   2  15.52   3  15.52   4  15.52  5  15.52  6  15.52  7  15.52  8  15.52
   9  15.52  10  15.52  11  15.52  12  15.52  13  15.52  14  15.52
NODE # AND TEMP AT INF.  1  16.00   2  16.00   3  16.00   4  16.00  5  16.00  6  16.00  7  16.00  8  16.00
   9  16.00  10  16.00  11  16.00  12  16.00  13  16.00  14  16.00
NODE # AND TEMP AT INF.  1  16.48   2  16.48   3  16.48   4  16.48  5  16.48  6  16.48  7  16.48  8  16.48
   9  16.48  10  16.48  11  16.48  12  16.48  13  16.48  14  16.48
NODE # AND TEMP AT INF.  1  16.96   2  16.96   3  16.96   4  16.96  5  16.96  6  16.96  7  16.96  8  16.96
   9  16.96  10  16.96  11  16.96  12  16.96  13  16.96  14  16.96
NODE # AND TEMP AT INF.  1  17.44   2  17.44   3  17.44   4  17.44  5  17.44  6  17.44  7  17.44  8  17.44
   9  17.44  10  17.44  11  17.44  12  17.44  13  17.44  14  17.44
NODE # AND TEMP AT INF.  1  17.92   2  17.92   3  17.92   4  17.92  5  17.92  6  17.92  7  17.92  8  17.92
   9  17.92  10  17.92  11  17.92  12  17.92  13  17.92  14  17.92
```

Figure C-19. (continued)

```
NODE # AND TEMP AT INF.  1  18.40   2  18.40   3  18.40   4  18.40  5  18.40  6  18.40  7  18.40  8  18.40
   9  18.40  10  18.40  11  18.40  12  18.40  13  18.40  14  18.40
NODE # AND TEMP AT INF.  1  18.88   2  18.88   3  18.88   4  18.88  5  18.88  6  18.88  7  18.88  8  18.88
   9  18.88  10  18.88  11  18.88  12  18.88  13  18.88  14  18.88
NODE # AND TEMP AT INF.  1  19.36   2  19.36   3  19.36   4  19.36  5  19.36  6  19.36  7  19.36  8  19.36
   9  19.36  10  19.36  11  19.36  12  19.36  13  19.36  14  19.36

TEMPERATURE DISTRIBUTION AT TIME STEP     10.000
10.69  10.70  10.76  10.91  11.38  12.20  13.50  14.99  16.49  17.48  17.25  16.63  15.86  15.15  14.87
 9.82   9.82   9.81   9.77   9.67   9.52   9.39   9.46   9.93  10.67  10.49  10.03   9.69   9.52   9.48
10.04  10.04  10.04  10.04  10.06  10.07  10.07  10.02   9.91   9.84   9.86   9.90   9.96  10.01  10.02
 9.98   9.99   9.99   9.99   9.99   9.99   9.99  10.00  10.02  10.03  10.02  10.02  10.01  10.00  10.00
10.01  10.00  10.00  10.00  10.00  10.00  10.00  10.00  10.00   9.99  10.00  10.00  10.00  10.00  10.00
10.00  10.00  10.00  10.00  10.00  10.00  10.00  10.00  10.00  10.00  10.00  10.00  10.00  10.00  10.00
10.01  10.01  10.01  10.00  10.00  10.00  10.00  10.00  10.00  10.00  10.00  10.00  10.00  10.00  10.00
10.01  10.01  10.02  10.02  10.01  10.01  10.00  10.00  10.00  10.02  10.02  10.00  10.00  10.00  10.00
10.00  10.00  10.00  10.00  10.00  10.00  10.00  10.00  10.02   9.98   9.98  10.01  10.00  10.00  10.00
10.00  10.00  10.00  10.00  10.00   9.99  10.00   9.99  10.04   9.92   9.92  10.03   9.99  10.00  10.00
10.00  10.00  10.00  10.00  10.00  10.00  10.00  10.00  10.00  10.03  10.03  10.00  10.00  10.00  10.00
10.00  10.00  10.00  10.00  10.00  10.00  10.00  10.00  10.00   9.99   9.99  10.00  10.00  10.00  10.00
10.00  10.00  10.00  10.00  10.00  10.00  10.00  10.00  10.00  10.00  10.00  10.00  10.00  10.00  10.00
10.00  10.00  10.00  10.00  10.00  10.00  10.00  10.00  10.00  10.00  10.00  10.00  10.00  10.00  10.00


B. FLUX DISCHARGE                  .000000                  .000000
CHANGE IN STORAGE                  .620931+12               .794988+11
QUANTITY PUMPED                   -.755490+12              -.754061+11
DIFFERENCE                        -.471091+07              -502208.
                                 CUMULATIVE MASS BLANCE    RATES FOR THIS TIME STEP
CONDUCTIVE TRANSFER                .676988+11               .759752+10
CONVECTIVE TRANSFER--OUT           .000000                  .000000
CONVECTIVE TRANSFER--IN            .130872+13               .147307+12
B. FLUX RECHARGE                   .000000                  .000000
```

Figure C-19. Program output for Mohawk River problem.

The Columbia Generating Station Site

This example is representative of the application described in section 5 of this report in which temperatures and water flows in the alluvial aquifer along the Wisconsin River were simulated after construction of a cooling lake on the river flood plain. Temperatures and water flows were simulated in an aquifer cross section shown in the schematic diagram in Figure C-20, and a finite element grid was used to discretize the cross section (Figure C-21). The parameters, boundary conditions, and initial conditions used in the simulation are listed in Table C-5.

The input data and the program output for a 9-day simulation are listed in Figure C-22 and Figure C-23. ENTRY BOUND was called at each time step to change the temperature of the water in the cooling lake and the air temperature.

TABLE C-5. PARAMETERS, INITIAL CONDITIONS, AND BOUNDARY CONDITIONS USED IN THE COLUMBIA GENERATING STATION PROBLEM

Water flow	
Boundary conditions	
River	Specified head at 778 ft[a]
Marsh	Specified head at 780 ft
Lake	Specified head at 789.2 ft
All other boundaries	No flow
Parameters	
K_{11}, K_{22}	Ten sets of values specified (refer to program output)
Heat flow	
Initial conditions	$T_i = 10^\circ C$
Boundary conditions	
Lake	Convective flux in
Marsh, River	Convective flux out and conductive flux
All other boundaries	No flow
Parameters	
K^t_{11}, K^t_{22}, ρC_s	Ten sets of values specified (refer to program output)
α_L	0.33 ft
α_T	0.08 ft
Time step	3 days

---

[a] The units of feet, gallons, calories, and days were used in this simulation.

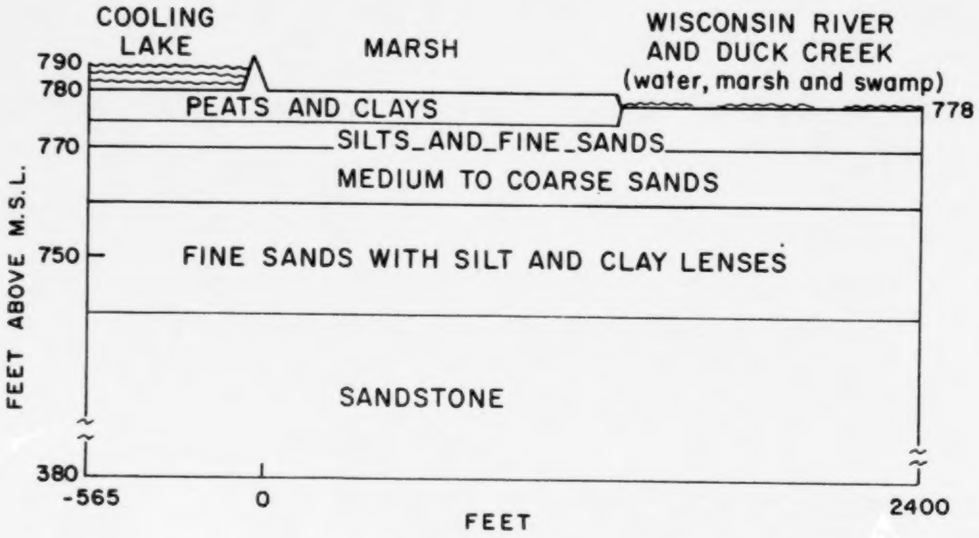

COOLING LAKE
MARSH
WISCONSIN RIVER AND DUCK CREEK (water, marsh and swamp)
790
780
778
770
750
380
PEATS AND CLAYS
SILTS AND FINE SANDS
MEDIUM TO COARSE SANDS
FINE SANDS WITH SILT AND CLAY LENSES
SANDSTONE
FEET ABOVE M.S.L.
-565
0
2400
FEET

Figure C-20. Schematic cross section of the Columbia Generating Station site along an east-west line.

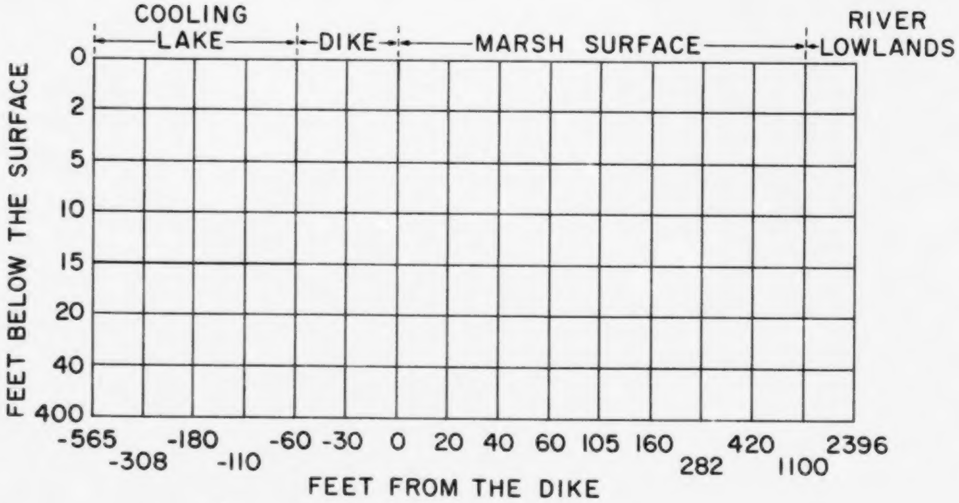

COOLING LAKE
DIKE
MARSH SURFACE
RIVER LOWLANDS
FEET BELOW THE SURFACE
0
2
5
10
15
20
40
400
-565
-308
-180
-110
-60
-30
0
20
40
60
105
160
282
420
1100
2396
FEET FROM THE DIKE

Figure C-21. The grid used to discretize the cross section simulated at the Columbia Generating Station site.

```
CGS  130'S
   UNITS USED ARE   GALLONS FOR FLOW, FT FOR DISTANCE, CAL, CENTIGRADE
105 128 10
3 0 0 0 1 1E14
(1X,8G12.6)

(1X,7G12.6)

GROUP 22
16 8
-565 -308 -180 -110 -60 -30 0 20 40 60 105 160 260 420 1100 2396
0 -2 -5 -10 -15 -20 -40 -400
GROUP III
3 1 1 3785.68
10 0 1
0 -1 1
89.3 0 0 0 0 0 0 0 89.3 0 0 0 0 0 0 0 89.3 0 0 0 0 0 0 0
89.3 0 0 0 0 0 0 0 89.3 0 0 0 0 0 0 0 84.5 0 0 0 0 0 0 0
80 0 0 0 0 0 0 0 80 0 0 0 0 0 0 0 80 0 0 0 0 0 0 0 80 0 0 0 0 0 0 0
80 0 0 0 0 0 0 0 80 0 0 0 0 0 0 0 80 0 0 0 0 0 0 0 79 0 0 0 0 0 0 0
78.5 0 0 0 0 0 0 0 78.5 0 0 0 0 0 0 0 999
0 0 1
GROUP IV CGS 130'S CARDS USED FOR SAVANNAH PAPER
10
1.6 1.6 1 .7 .7 1 .2 .2
1 -1  1
1 2 3 4 4 6 5
1 2 3 4 4 6 5
1 2 3 4 4 6 5
1 2 3 4 4 6 5
7 2 3 4 4 6 5
7 2 10 8 8 6 5
1 9 10 8 8 6 5
1 9 10 8 8 6 5
1 9 10 8 8 6 5
1 9 10 8 8 6 5
1 9 10 8 8 6 5
1 9 10 8 8 6 5
1 9 10 8 8 6 5
1 1 10 8 8 6 5
1 1 10 8 8 6 5           999
1
100 20 0 20000 20000 26000 .33 .08
2
1  1  0 8000 8000 26000 .33 .08
3
```

(continued)

Figure C-22. The data deck used to model heat flow at the Columbia Generating Station site.

```
100 20 0 17000 17000 21000 .33 .08
4
500 50 0 17000 17000 21000 .33 .08
5
10 10 0 8000 8000 26000 .33 .08
6
63 1 72 1 81 1 144 1 153 1 162 1
63 1 72 1 81 1 144 1 153 1 162 1
0 0
GROUP VI
0
GROUP VII
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 00 0000 00 0
0 0
GROUP VIII
0 9
1 3 2 3 3 3 4 3 5 3 6 3 7 3 8 3 9 3 10 3 11 3
GROUP IX
1 1 40 1 0 0
2 9 16 24 26 33 40 48 50 57 64 72 74 81 88 96 98 105 112 120 122 129 136 144 146
153 160 168 170 177 184 192 194 201 208 216 218 225 232 240
-1.0001 1 1 1 0 0
1 1 1 1 1 1 1 1 11 1 15            5
1 860
             857    10.2222        24.6111
             858    12.5000        26.0555
             859    12.7222        25.1666
             860    11.8333        24.6111
1 1 1 1 1 1 1 0
-81 1
-142 -284 -142 142 284 142
0 0 0 3.15E6 6.36E6 3.15E6
12000 12000 12000 12000 12000 12000 12000 12000
12000 12000 12000 12000 12000 12000 12000 12000
0 0
0 0 0 0 0 0
0 0 0 0 0 0 0
12000 12000 12000 12000 12000 12000 12000 12000
12000 12000 12000 12000 12000 12000 12000 12000
81 1
561.5 1123 561.5 -561.5 -1123 -561.5
-9.08E6 -1.82E7 -9.08E6 0 0 0
12000 12000 12000 12000 12000 12000 12000 12000
12000 12000 12000 12000 12000 12000 12000 12000
0 0
0 0 0 0 0 0 0
0 0 0 0 0 0 0
12000 12000 12000 12000 12000 12000 12000 12000
12000 12000 12000 12000 12000 12000 12000 12000
```

Figure C-22. The data deck used to model heat flow at the Columbia Generating Station site.

```
  CGS 130'S  UNITS USED ARE   GALLONS FOR FLOW.FT FOR DISTANCE.CAL.CENTIGRADE
# OF NODES 128  # OF ELEMENTS 105   PANDWIDTH  10

VELOCITIES ARE BEING CALCULATED AT CURRENT TIME STEP   1

THE X-SPACING IS

 -565.0000    -508.0000    -160.0000    -110.0000     -60.0000     -30.0000       0.0          20.0000
   40.0000      60.0000     105.0000    -160.0000     260.0000     420.0000    1100.0000     2396.0000

THE Y-SPACING IS

    0.0         -4.0000      -5.0000     -10.0000     -15.0000     -20.0000     -40.0000     -400.0000


EDUCATION 2 TIME STEP  3.00000        HEAT OR MASS CAPACITY COEFFICIENT  3785.68


PARAMETER FACTORS--IN ORDER    1.60000    1.60000    1.00000    .700000    .700000    1.00000    .200000    .200000

MATERIAL    HOR PERM     VER PERM     STORAGE     HOR THERM COND    VERT THERM COND    SPECIFIC DISPERSION COEFS
                                                                                   HEAT
             1  160.0      32.00        .0000        .1400+05          .1400+.05         .2600+05  .6600-01  .1600-01

             2  1.800      1.600        .0000        5600.             5600.             .2600+05  .6600-01  .1600-01

             3  160.0      32.00        .0000        .1190+05          .1190+05          .2100+05  .6600-01  .1600-01

             4  800.0      80.00        .0000        .1190+05          .1190+05          .2100+05  .6600-01  .1600-01

             5  16.00      16.00        .0000        5600.             5600.             .2600+05  .6600-01  .1600-01

             6  200.0      19.20        .0000        .1190+05          .1190+05          .2100+05  .6600-01  .1600-01

             7  1.800      1.600        .0000        5600.             5600.             .2600+05  .6600-01  .1600-01

             8  1600.      160.0        .0000        .1190+05          .1190+05          .2100+05  .6600-01  .1600-01

             9  3.200      3.200        .0000        5600.             5600.             .2600+05  .6600-01  .1600-01

            10  280.0      40.00        .0000        .1190+05          .1190+05          .2100+05  .6600-01  .1600-01
```

Figure C-23. Program output for Columbia Generating Station problem. (continued)

```
THE VALUES IN THE TYPE OF MATERIAL MATRIX ARE
  1.00000    2.00000    3.00000    4.00000    4.00000    6.00000    5.00000
  1.00000    2.00000    3.00000    4.00000    4.00000    6.00000    5.00000
  1.00000    2.00000    3.00000    4.00000    4.00000    6.00000    5.00000
  1.00000    2.00000    3.00000    4.00000    4.00000    6.00000    5.00000
  7.00000    2.00000    3.00000    4.00000    4.00000    6.00000    5.00000
  7.00000    2.00000    10.0000    8.00000    8.00000    6.00000    5.00000
  1.00000    9.00000    10.0000    8.00000    8.00000    6.00000    5.00000
  1.00000    2.00000    10.0000    8.00000    8.00000    6.00000    5.00000
  1.00000    9.00000    10.0000    8.00000    8.00000    6.00000    5.00000
  1.00000    9.00000    10.0000    8.00000    8.00000    6.00000    5.00000
  1.00000    9.00000    10.0000    8.00000    8.00000    6.00000    5.00000
  1.00000    9.00000    10.0000    8.00000    8.00000    6.00000    5.00000
  1.00000    9.00000    10.0000    8.00000    8.00000    6.00000    5.00000
  1.00000    1.00000    10.0000    8.00000    8.00000    6.00000    5.00000
  1.00000    1.00000    10.0000    8.00000    8.00000    6.00000    5.00000


INFORMATION FOR MASS OR HEAT TRANSFER ACROSS A SPECIFIED HEAD BOUNDARY
ELEMENT NUMBER--BOUNDARY CODE--TEMPERATURE OR CONCENTRATION OF INCOMING FLUID
    1  1    .00000         8  1   .00000       15  1   .00000       22  1    .00000     29  1    .00000

   36  1
CONVECTIVE OUT AND CONDUCTIVE BOUNDARY INFORMATION                 ELEM #--BOUNDARY CODE--TRANSFER COEFFICIENT
    1 1  12000.0        8 1  12000.00        15 1  12000.0          22 1  12000.0        43 1  .000000
  50 1  .000000       57 1  .000000        64 1  .000000          71 1  .000000        78 1  .000000
  85 1  .000000       92 1  .000000        99 1  .000000          50 1  8000.00        57 1  8000.00
  64 1  8000.00       71 1  8000.00        75 1  8000.00          85 1  8000.00        92 1  8000.00
  99 1  8000.00


WATER FLOW BOUNDARIES--ELEM # AND BOUNDARY CODE
   1  1       8  1      15  1      22  1      29  1      36  1      43  1      50  1      57  1      64  1
  71  1      78  1      85  1      92  1      99  1

POTENTIAL DISTRIBUTION AT TIME STEP     .00000
  89.3000    89.3805    88.70000   88.6525    88.6355    88.6218    88.4519    86.8035
  89.3000    89.2615    88.3858    87.9897    87.9564    87.9318    87.6714    85.9030
  89.3000    89.2224    86.9984    85.8184    86.7581    86.7171    86.3573    84.9464
  89.3000    89.2390    85.9284    85.6591    85.5716    85.5204    85.2024    84.4145
  89.3000    89.0895    84.7128    84.3900    84.3048    84.2692    84.2345    84.0487
  84.5000    89.8503    83.3893    84.3395    83.3358    83.3626    83.6783    83.8094
  80.0000    80.2692    82.5679    82.7641    82.8003    82.8165    83.1117    83.5782
```

Figure C-23. (continued)

```
-51.5399    -39.3041   -31.2270    -22.3388    -10.7826    -4.78335    -.416066
-31.6491    -34.3912   -33.3403    -24.6573    -12.7486    -4.72700    -.530258
-30.5509    -29.3358   -28.9887    -22.8616    -12.2211    -4.62473    -.619473
-53.6430    -54.0320   -52.3883    -41.7264    -23.2787    -9.53972    -1.59924
-48.1144    -45.1488   -44.7073    -35.9204    -20.6327    -9.53262    -2.14461
-34.1526    -38.6777   -33.9313    -20.7011    -19.2395    -11.5253    -3.85953
-35.987     -15.4392   -35.8504    -31.1358    -22.1957    -14.5765    -5.71147
-134.856    -156.379   -129.908    -105.243    -64.2157    -34.1659    -12.2147
 12.9762     18.4182    12.3757     9.10431     3.91385     .408686    -.678204

THE DISPERSION ROUTINE IS BEING USED
MODE # AND TEMP AT INF.    1   25.76    8   25.76    15   25.76    22   25.76    43   25.76    50   25.76    57    9.00    64    9.00
  71    9.00    78    9.00    85    9.00   92    9.00    99    9.00    50    9.00    57    9.00    64    9.00
  71    9.00    78    9.00    35    9.00   92    9.00    99    9.00


TEMPERATURE DISTRIBUTION AT TIME STEP    .00000
  23.0526     10.6176     9.98973     10.0160     9.99720     10.0002     9.99999     10.0000
  24.0723     10.8383     9.88251     10.0196     9.99675     10.0002     9.99999     10.0000
  23.2023     11.3082     9.35211     10.0222     9.99657     10.0002     9.99999     10.0000
  28.3750     12.0244     9.32307     10.0226     9.99680     10.0002     9.99999     10.0000
  28.4317     11.7903     9.80178     10.0184     9.99758     10.0001    10.00000     10.0000
  9.36313     9.22293     10.8940     10.0030     9.98476     10.0030     10.0000    10.00000
  10.6118     10.1791     9.97268     10.9030     9.99928     10.0001    10.00000     10.0000
  9.68002     9.95359     10.0947     9.99947     10.0001     10.0000    10.00000    10.00000
  9.38139     10.0437     9.99198     10.0016     9.99978     10.0000     10.0000     10.0000
  9.48225     10.0148     9.99649     10.0011     9.99965     10.0001    10.00000     10.0000
  9.42817     10.0091     9.99830     10.0004     9.99997     9.99996     10.0000    10.00000
  9.44609     10.3090     10.6900     10.0791     10.0001     9.99995     10.0000    10.00000
  9.35584     9.97909     10.8035     9.99838     10.0001     9.99998     10.0000    10.00000
  9.43319     9.99364     10.0908     9.99995    10.00000     9.99996     10.0000    10.00000
  9.38966     9.98055     10.0016     9.99975     10.0001     9.99997     10.0000    10.00000
  9.40101     9.98236     10.0016     9.99369     10.0001     9.99998     10.0000    10.00000

                          CUMULATIVE MASS BLANCE       RATES FOR THIS TIME STEP
CONDUCTIVE TRANSFER              .114876+09                 .114876+09
CONVECTIVE TRANSFER--OUT         .446673+08                 .448673+08
CONVECTIVE TRANSFER--IN          .118618+09                 .112618+09
B. FLUX RECHARGE                 .000000                    .000000
B. FLUX DISCHARGE                .000000                    .000000
CHANGE IN STORAGE                .188595+09                 .188595+09
QUANTITY PUMPED                  .000000                    .000000
DIFFERECE                        32501.0                    32501.0
QXQT
```

Figure C-23. Program output for Columbia Generating Station problem.

```
 80.0000     80.1100     82.1493     82.4126     82.4586     82.4821     82.7735     83.4091
 80.0000     80.1228     81.6793     82.1055     82.1513     82.1748     82.4630     83.2320
 80.0000     80.1019     81.6384     81.8778     81.8771     81.3987     82.1775     83.0495
 80.0000     80.0735     81.1874     81.3707     81.3596     81.3761     81.6172     82.8280
 80.0000     80.0499     80.7483     80.8439     80.5631     80.8744     81.0584     82.1134
 80.0000     80.0004     80.1561     80.1600     80.1619     80.1647     80.2634     81.2531
 79.0000     79.0329     79.0889     79.1506     79.1630     79.1703     79.2952     80.1966
 78.5000     78.4972     79.4910     78.4856     78.4846     78.4842     78.4326     78.5328
 78.5000     78.5814     79.0047     78.5070     78.5075     78.5077     78.5085     78.4860
                        CUMULATIVE MASS BLANCE              RATES FOR THIS TIME STEP
B. FLUX RECHARGE               417.916                          417.916
B. FLUX DISCHARGE              417.716                          417.916
CHANGE IN STORAGE              .000000                          .000000
QUANTITY PUMPED                .000000                          .000000
DIFFERECE                      .200280                          .200280


TIME STEP    .000000
FLOWS IN THE X DIPRECTION
-.996091-02 -.502178-02 -1.66834     -8.87056     -9.05049     -9.72060     -16.0007
-.414756-01 -.179398-01 -5.99535     -31.4491     -32.0244     -33.5617     -43.3976
-.281315-01 -.315544-01 -10.8207     -56.9305     -57.8404     -57.0730     -58.9506
-.378952    -.552483-01 -16.8848     -86.1615     -85.5518     -75.3980     -65.2530
-.453898    -.444803    -26.8867     -114.767     -106.215     -82.8756     -64.8656
-.366091    -.499161    -27.6845     -105.815     -122.489     -63.0097     -65.0647
-1.08199    -.117809    -22.8950     -117.353     -114.864     -57.1372     -62.0496
 .372479-01 -.524130-01 -17.1250     -104.200     -104.412     -52.4734     -59.6368
-.141957    -.533644-01 -1511524     -92.2431     -93.4833     -47.7045     -57.2477
-.359611-01 -.434412-01 -12.6539     -77.3605     -78.5292     -40.8782     -53.3715
-.532992-01 -.343201-01 -10.0149     -60.7450     -61.6645     -32.7559     -47.7426
-.673201-01 -.251925-01 -7.56603     -47.0623     -47.9377     -25.5635     -40.4953
-1.67195    -.515699-01 -7.71698     -42.0485     -42.3296     -20.8387     -30.9582
-.208530    -.339984    -1.10422     -6.71201     -6.81793     -3.74431     -8.90867
 .443338-03  .275349-02  .159093-01   .115948      .121541      .647594-01  -.394883-01

FLOWS IN THE Y DIRECTION
 101.654     102.223     100.305      87.9404      66.8651      45.0807      18.5753
 100.988     98.5690     96.0592      81.4697      57.0099      32.3655      7.68152
 80.2117     87.2790     85.4840      70.3572      43.8252      19.3443      2.90531
 109.236     86.4973     80.4588      58.7120      29.4592      7.18937      .918946
 8.97147     32.7361     30.5889      18.1331      1.79816     -3.43804      .399501-01
 3.87355    -12.4882    -14.9164     -13.2480     -17.5263     -7.47321     -.338393
```

Figure C-23. (continued)

The Heat Pump Problem

This problem was coded to study the impact of the injection of cooled waters from a heat pump into a shallow ground-water aquifer. The problem is discussed in more detail by Andrews (1978).

The finite element grid used to discretize the problem is shown in Figure C-24. Since the problem is symmetric, only half the system was modeled. A novel feature of the finite element grid is that the thickness of each element is proportional to distance along the horizontal axis from the wells--a quasi-radial formulation. The program input data and a sample program output are listed in Figure C-25 and Figure C-26.

A few unusual features are incorporated into this problem. The heat pump problem was programmed to recompute flows at odd intervals, corresponding to the end of each of the seasons. The intervals at which flows were recomputed are listed on cards 3 and 4 of data group IX. ENTRY CHAN was called each time flows were recomputed to read in the pumping schedule for the current interval. Air temperatures for the entire simulation period were read in by a call to ENTRY LAKE at the beginning of program execution.

The major subroutines and the tasks performed by each are listed in a program flow chart (Figure C-27).

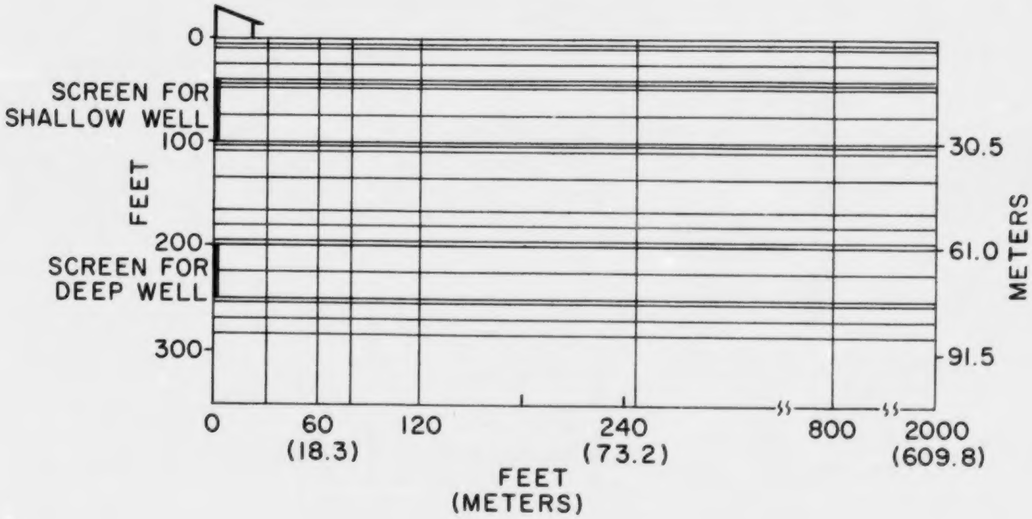


Figure C-24. Grid used to discretize the aquifer simulated in the heat pump problem.

```
  HOME HEATING AND COOLING--SYMMETRICAL SET-UP
 UNITS  FEET GALLONS DAYS CALORIES
168  198  11
3 0 0 0 1 1E17
(1x,9(G12.6)        )

(1x,8(G12.6)        )

GROUP II
22 9
0 2 10 25 40 48 50 75 100 102 110 135 165 180 195 200 225 250 255 270 285 350
2000 800 250 160 120 80 60 30 0 -30 -300
GROUP III
15 1 0 3785.68
10 0 1
0 -1 1
10 10 10 10 10 10 10 10 10
    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
                  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
999
0 -1 1
1 1 1 1 1 1 1 1 1 10 1 0 0 0 0 0 0 0 10 0 0 0 0 0 0 0 0
10 0 0 0 0 0 0 0 0 10 0 0 0 0 0 0 0 0 10 0 0 0 0 0 0 0 0
10 0 0 0 0 0 0 0 0 10 0 0 0 0 0 0 0 0 10 0 0 0 0 0 0 0 0
10 0 0 0 0 0 0 0 0 10 0 0 0 0 0 0 0 0 10 0 0 0 0 0 0 0 0
10 0 0 0 0 0 0 0 0 10 0 0 0 0 0 0 0 0 10 0 0 0 0 0 0 0 0
10 0 0 0 0 0 0 0 0 10 0 0 0 0 0 0 0 0 10 0 0 0 0 0 0 0 0
10 0 0 0 0 0 0 0 0 10 0 0 0 0 0 0 0 0 10 0 0 0 0 0 0 0 0
10 0 0 0 0 0 0 0 0 999
GROUP IV
4
1 1 1 .7 .7 1 1 1
0 -1 1
1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3
4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4   999
1
.1 .1 0 8000 8000 17000 10 1
2
10 100 0 17000 17000 21000 10 1
3
25 250 0 17000 17000 21000 10 1
4
10 100 0 17000 17000 21000 10 1
GROUP V
0 0
6 6
125 12 0 17000 17000 21000 .33 .08
7
1 1 0 8000 8000 26000 .33 .08
8
```

(continued)

Figure C-25. The data deck used to model the heat pump simulation with no regional groundwater flow.

```
1000 100 0 17000 21000 .33 .08
9 2 2 0 8000 8000 26000 .33 .08
10
175 25 0 17000 17000 21000 .33 .08
GROUP V
0 0
0 0
0 0
GROUP VI
0
GROUP VII
21
1 1 12000 8 1 12000 15 1 12000 22 1 12000
43 1 0 50 1 0 57 1 0 64 1 0 71 1 0 78 1 0 85 1 0 92 1 0 99 1 0
50 1 8000 57 1 8000 64 1 8000 71 1 8000 78 1 8000 85 1 8000 92 1 8000 99 1 8000
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
6
1 1 8 1 15 1 22 1 29 1 36 1
0 0 0 0 0 0 0
GROUP VIII
15 0
1 1 8 1 15 1 22 1 29 1 36 1 43 1 50 1 57 1 64 1 71 1 78 1 85 1 92 1 99 1
GROUP IX
1 0 0 0 0 8
53 69 93 101 52 68 92 100
1 140 140 1 140 140 1 140 140 1 3
5
10 860
     1.0000     29.5000    41.0000
     2.0000     28.5000    35.9000
     3.0000     27.5000    30.8000
1 1 1 1 1 1 1 0
```

Figure C-25. The data deck used to model the heat pump simulation with no regional groundwater flow.

```
          HOME HEATING AND COOLING--SYMMETRICAL SET-UP
          UNITS FEET GALLONS DAYS CALORIES

                                                                   04/07/78
                                                                   17:02:33

# OF NODES 198   # OF ELEMENTS 168   BANDWIDTH 11

VELOCITIES ARE BEING CALCULATED AT CURRENT TIME STEP     1

THE X-SPACING IS
   0.0         2.0000      10.0000     25.0000     40.0000     48.0000     50.0000     75.0000
 100.0000    102.0000     110.0000    135.0000    165.0000    180.0000    195.0000    200.0000
 225.0000    250.0000     255.0000    270.0000    285.0000    350.0000

THE Y-SPACING IS
 2000.000    800.0000     250.0000    160.0000    120.0000     80.0000     60.0000     30.0000
   0.0

EQUATION 2 TIME STEP  15.0000          HEAT OR MASS CAPACITY COEFFICIENT  3785.68

PARAMETER FACTORS--IN ORDER   1.00000   1.00000   1.00000   .700000   .700000   1.00000   1.00000   1.00000

  MATERIAL   HOR PERM    VER PERM    STORAGE   HOR THERM COND   VERT THERM COND   SPECIFIC   DISPERSION   COEFS
                                                                                HEAT
          1  .1000+00    .1000+00    .0000       5600.            5600.           .1700+05   10.00        1.000

          2  10.00       100.0       .0000      .1190+05         .1190+05         .2100+05   10.00        1.000

          3  25.00       250.00      .0000      .1190+05         .1190+05         .2100+05   10.00        1.000

          4  10.00       100.0       .0000      .1190+05         .1190+05         .2100+05   10.00        1.000

THE VALUES IN THE TYPE OF MATERIAL MATRIX ARE
  1.00000    1.00000    1.00000    1.00000    1.00000    1.00000    1.00000    1.00000
  2.00000    2.00000    2.00000    2.00000    2.00000    2.00000    2.00000    2.00000
  2.00000    2.00000    2.00000    2.00000    2.00000    2.00000    2.00000    2.00000
  3.00000    3.00000    3.00000    3.00000    3.00000    3.00000    3.00000    3.00000
  4.00000    4.00000    4.00000    4.00000    4.00000    4.00000    4.00000    4.00000
  4.00000    4.00000    4.00000    4.00000    4.00000    4.00000    4.00000    4.00000

  4.00000    4.00000    4.00000    4.00000    4.00000    4.00000    4.00000    4.00000
  4.00000    4.00000    4.00000    4.00000    4.00000    4.00000    4.00000    4.00000
  4.00000    4.00000    4.00000    4.00000    4.00000    4.00000    4.00000    4.00000
```

Figure C-26. Program output for the heat pump problem. (continued)

```
   4.00000     4.00000     4.00000     4.00000     4.00000     4.00000     4.00000     4.00000
   4.00000     4.00000     4.00000     4.00000     4.00000     4.00000     4.00000     4.00000
   4.00000     4.00000     4.00000     4.00000     4.00000     4.00000     4.00000     4.00000
   4.00000     4.00000     4.00000     4.00000     4.00000     4.00000     4.00000     4.00000
   4.00000     4.00000     4.00000     4.00000     4.00000     4.00000     4.00000     4.00000
   4.00000     4.00000     4.00000     4.00000     4.00000     4.00000     4.00000     4.00000
   4.00000     4.00000     4.00000     4.00000     4.00000     4.00000     4.00000     4.00000
   4.00000     4.00000     4.00000     4.00000     4.00000     4.00000     4.00000     4.00000
   4.00000     4.00000     4.00000     4.00000     4.00000     4.00000     4.00000     4.00000
   4.00000     4.00000     4.00000     4.00000     4.00000     4.00000     4.00000     4.00000
   4.00000     4.00000     4.00000     4.00000     4.00000     4.00000     4.00000     4.00000
   4.00000     4.00000     4.00000     4.00000     4.00000     4.00000     4.00000     4.00000

POINT SOURCES OF WATER--NODE # AND AMOUNT
  63    1.00000          72    1.00000          81    1.00000         144    1.00000         153    1.00000
 162    1.00000

POINT SOURCES OF HEAT
  63    1.00000          72    1.00000          81    1.00000         144    1.00000         153    1.00000
 162    1.00000
CONVECTIVE OUT AND CONDUCTIVE BOUNDARY INFORMATION           ELEM #--BOUNDARY CODE--TRANSFER  COEFFICIENT
    0 0  .00000

HEAT FLOW BOUNDARIES--NODE # AND BOUNDARY CODE
  1  3        2  3        3  3        4  3        5  3        6  3        7  3        8  3        9  3

TEMPERATURE DISTRIBUTION AT TIME STEP     .250000
  10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000
  10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000
  10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000
  10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000
  10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000
  10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000
  10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000
  10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000
  10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000
  10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000
  10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000
  10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000
  10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000
  10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000
  10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000
  10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000
  10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000
```

Figure C-26. (continued)

```
 10.0000  10.0000  10.0000  10.0000  10.0000  10.0000  10.0000  10.0000  10.0000
 10.0000  10.0000  10.0000  10.0000  10.0000  10.0000  10.0000  10.0000  10.0000
 10.0000  10.0000  10.0000  10.0000  10.0000  10.0000  10.0000  10.0000  10.0000
 10.0000  10.0000  10.0000  10.0000  10.0000  10.0000  10.0000  10.0000  10.0000
 10.0000  10.0000  10.0000  10.0000  10.0000  10.0000  10.0000  10.0000  10.0000

POTENTIAL DISTRIBUTION AT TIME STEP     .000000
 10.00000 10.00000 10.00000 10.00000 10.00000 10.00000 10.00000 10.00000 10.00000
 10.0013  10.0011  9.98874  9.97939  9.97459  9.96998  9.96806  9.96583  9.96433
 10.0014  10.0012  9.98832  9.97842  9.97322  9.96813  9.96599  9.96346  9.96173
 10.0015  10.0013  9.98773  9.97589  9.96883  9.96082  9.95687  9.95193  9.94858
 10.0015  10.0014  9.98772  9.97457  9.96588  9.95443  9.94715  9.93387  9.91940
 10.0015  10.0015  9.98799  9.97279  9.96126  9.94321  9.92931  9.89177  9.78885
 10.0015  10.0015  9.98807  9.97238  9.96016  9.94052  9.92456  9.88408  9.72716
 10.0016  10.0018  9.98989  9.97052  9.95306  9.92129  9.89284  9.81196  9.52379
 10.0017  10.0022  9.99331  9.97712  9.96360  9.94163  9.92412  9.88125  9.72221
 10.0017  10.0022  9.99365  9.97797  9.96504  9.94436  9.92863  9.88812  9.78247
 10.0017  10.0023  9.99511  9.98172  9.97125  9.95580  9.94514  9.92506  9.90286
 10.0018  10.0026  10.0005  9.99548  9.99222  9.98861  9.98716  9.98618  9.98653
 10.0019  10.0029  10.0075  10.0129  10.0164  10.0203  10.0221  10.0242  10.0256
 10.0019  10.0031  10.0109  10.0215  10.0291  10.0391  10.0449  10.0528  10.0565
 10.0020  10.0033  10.0139  10.0295  10.0419  10.0611  10.0753  10.1086  10.1655
 10.0020  10.0034  10.0148  10.0319  10.0458  10.0683  10.0860  10.1293  10.2876
 10.0020  10.0037  10.0183  10.0393  10.0557  10.0903  10.1192  10.2003  10.4890
 10.0019  10.0041  10.0201  10.0383  10.0527  10.0755  10.0933  10.1367  10.2951
 10.0019  10.0042  10.0203  10.0373  10.0502  10.0698  10.0842  10.1176  10.1746
 10.0019  10.0044  10.0206  10.0336  10.0421  10.0527  10.0588  10.0669  10.0708
 10.0019  10.0046  10.0206  10.0298  10.0346  10.0396  10.0417  10.0442  10.0460
 10.0018  10.0051  10.0198  10.0222  10.0227  10.0229  10.0228  10.0227  10.0225

                         CUMULATIVE MASS BLANCE        RATES FOR THIS TIME STEP
B. FLUX RECHARGE               .000000                        .000000
B. FLUX DISCHARGE              .000000                        .000000
CHANGE IN STORAGE              .000000                        .000000
QUANTITY PUMPED                .000000                        .000000
DIFFERENCE                     .000000                        .000000
```

Figure C-26. Program output for the heat pump problem.

I. BRIEF DESCRIPTION OF THE PROGRAM ROUTINES

A. Main Routine

This routine controls the flow of the program and the program dimensions. The parameter variables L, N, M control the maximum number of elements, nodes, and bandwidth respectively.

1. Internal Subroutine ITERAT
   This subroutine controls the solution of a steady-state areal problem by an iterative method.
2. Internal Subroutine FLOADD
   This subroutine adds in a constant specified flow rate to the values calculated by the water-flow equation.

B. Subroutine DATAIN

This subroutine reads in the program data.

1. ENTRY DI
   This entry point passes arguments from the main routine to subroutine DATAIN.
2. Internal Subroutine ELREAD
   This subroutine reads in the element node numbers.
3. Internal Subroutine P
   This subroutine creates a rectangular grid and numbers the nodes and elements in the grid.

C. Subroutine STRUCT

This subroutine assembles the global structure matrix and the global capacitance matrix.

D. Subroutine GAUSS

This subroutine forms the element structure and capacitance matrices by Gaussian Quadrature.

1. ENTRY DERIVE
   The main entry point for subroutine GAUSS.
2. Internal Subroutine CONVEC
   This routine handles the convective boundaries.

E. Subroutine SHAPE1

This subroutine computes the shape functions for a linear element.

F. Subroutine SHAPE

This subroutine computes the shape functions for elements with one or more nonlinear sides.

G. Subroutine LOAD

This subroutine computes the global recharge matrix by adding in the various sources and sinks.

1. ENTRY HEATE
   The main entry point for subroutine LOAD.
2. Internal Subroutine CB
   This subroutine computes mass or heat flow across a specified potential boundary.

H. Subroutine SOLVE

This subroutine is a symmetric banded matrix equation solver.

1. ENTRY BACK
   Entry point for the back substitution of the recharge matrix.
2. ENTRY MULTI
   Entry point for the multiplication in a transient problem.

I. Subroutine ASOLVE

This subroutine is an assymmetric banded matrix equation solver.

1. ENTRY ABACK
   Entry point for the back substitution.
2. ENTRY AMULTI
   Entry point for matrix multiplication in a transient problem.

J. Subroutine BALAN

This subroutine computes a mass balance.

1. ENTRY MASBAL
   The main entry point for subroutine BALAN.
2. ENTRY BPRINT
   The entry point for the printing of the mass balance.
3. ENTRY WATER
   Entry point for computing water flows in each element.
4. ENTRY VELO
   Computes water velocities at the Gauss points.
5. ENTRY VCENT
   Computes water velocities at the center of each element.
6. Internal Subroutine FLOWW
   Subroutine for computing element flows.

K. Subroutine FLOWS

This subroutine prints out element flows and nodal values.

1. ENTRY FFLOW
   Entry point for the printing of nodal potentials on file 14.

2. ENTRY FFFLOW
   Entry point for the printing of nodal temperatures or concentrations on file 14.
3. ENTRY WFLOW
   Entry point for printing water flows for each element.
4. ENTRY HPRINT
   Entry point for printing nodal temperature or concentration values.
5. ENTRY WPRINT
   Entry point for printing nodal potential values.

L. Subroutine PARAM

This subroutine computes element parameters that are a function of velocity or temperature and localized coordinates.

1. ENTRY MECD
   Entry point for computing dispersivity.
2. ENTRY CORD
   Entry point for the computation of localized coordinates.
3. ENTRY PE
   Entry point for computing hydraulic conductivity as a function of temperature.
4. ENTRY PEE
   Entry point for the computation of aquifer thickness in an areal problem.

M. Subroutine BOUNDA

This routine is used to change parameter values or boundary conditions at each time step. Five entry points, ENTRY LAKE, ENTRY BVAL, ENTRY BOUND, ENTRY CHANG, ENTRY CHAN, are provided for user programming.

N. Subroutine EIGEN
This subroutine is used to compute the maximum stable time step for a transient problem.

O. Subroutine ADJUST

This subroutine moves the upper boundary in a cross-section problem.

1. ENTRY ADJUST
   The main entry point for subroutine ADJUST.

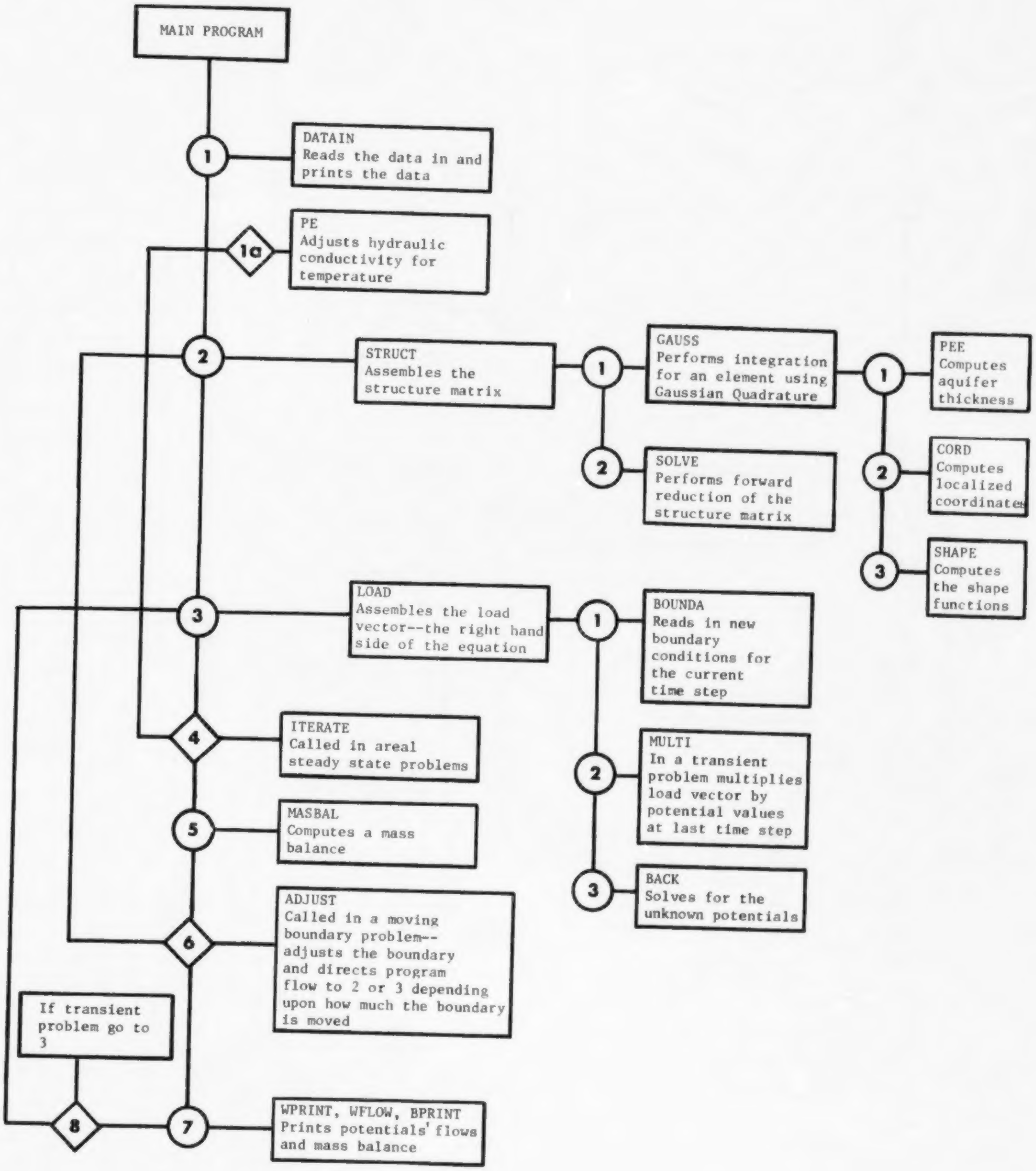

MAIN PROGRAM
1
DATAIN Reads the data in and prints the data
1a
PE Adjusts hydraulic conductivity for temperature
2
STRUCT Assembles the structure matrix
1
GAUSS Performs integration for an element using Gaussian Quadrature
1
PEE Computes aquifer thickness
2
CORD Computes localized coordinates
3
SHAPE Computes the shape functions
2
SOLVE Performs forward reduction of the structure matrix
3
LOAD Assembles the load vector--the right hand side of the equation
1
BOUNDA Reads in new boundary conditions for the current time step
2
MULTI In a transient problem multiplies load vector by potential values at last time step
3
BACK Solves for the unknown potentials
4
ITERATE Called in areal steady state problems
5
MASBAL Computes a mass balance
6
ADJUST Called in a moving boundary problem--adjusts the boundary and directs program flow to 2 or 3 depending upon how much the boundary is moved
If transient problem go to 3
8
7
WPRINT, WFLOW, BPRINT Prints potentials' flows and mass balance

Figure C-27. Program flow chart.

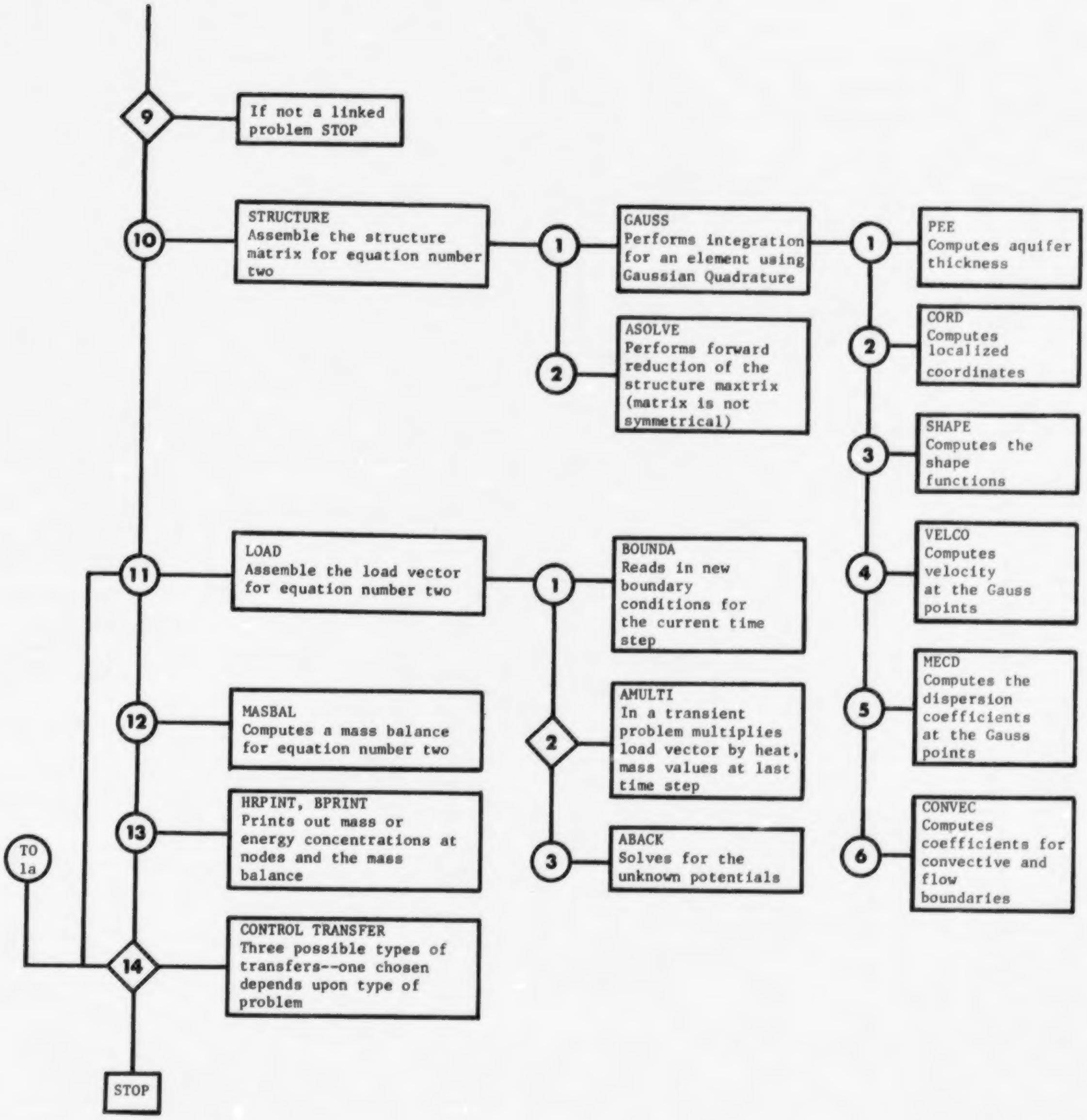

9
If not a linked problem STOP
10
STRUCTURE Assemble the structure matrix for equation number two
1
GAUSS Performs integration for an element using Gaussian Quadrature
1
PEE Computes aquifer thickness
2
CORD Computes localized coordinates
3
SHAPE Computes the shape functions
4
VELCO Computes velocity at the Gauss points
5
MECD Computes the dispersion coefficients at the Gauss points
6
CONVEC Computes coefficients for convective and flow boundaries
2
ASOLVE Performs forward reduction of the structure maxtrix (matrix is not symmetrical)
11
LOAD Assemble the load vector for equation number two
1
BOUNDA Reads in new boundary conditions for the current time step
2
AMULTI In a transient problem multiplies load vector by heat, mass values at last time step
3
ABACK Solves for the unknown potentials
12
MASBAL Computes a mass balance for equation number two
13
HRPINT, BPRINT Prints out mass or energy concentrations at nodes and the mass balance
TO 1a
14
CONTROL TRANSFER Three possible types of transfers--one chosen depends upon type of problem
STOP

Figure C-27 (continued)

THE FINITE ELEMENT PROGRAM LISTING

```
C
C********************************************************************
C
C          THE MAIN ROUTINE FOR THE FINITE ELEMENT PROGRAM
C                 BY CHARLES ANDREWS
C
C
C********************************************************************
C
C
C              THE PARAMETER STATEMENTS
C            L--NUMBER OF ELEMENTS, N--NUMBER OF NODES.
C            M-- BANDWIDTH,Z--SIZE OF ARRAYS CONTAINING SOURCE
C             SINK, AND BOUNDARY INFORMATION
C
                PARAMETER M=17,N=160,L=130,Z=50
C
C
C********************************************************************
                PARAMETER P=M*2-1,BBB=N
         INTEGER AY(4),AZ(4)
         DATA AY/4,1,4,3/
         DATA AZ/3,2,1,2/
        DOUBLE PRECISION S,T
        REAL MAT  INFLOW
         COMMON/ACC/KK(12).K1,K2,K3,K4
          COMMON/AM/NLA,PCT/CON/MC1,MC2,NCONV
       */CONT/LA,LB,LC,LD.LE,LF,LG LH
        COMMON/HI/TITLE(25),V(26),VV(26)
        COMMON/HM/PXX PYY,PXY.KAD/HH/LFLOW,LON/ME/MEQ
         COMMON/ATHICK/ASIZE,NTHICK.THICK ERROR
C  MODEL PARAMETERS--1
        DIMENSION WX(L),WY(L),STO(L),INFLOW(L)
C--MODEL PARAMETERS--2
        DIMENSION PX(L),PY(L),PCX(L),HEAT(L),DIFF(L,2)
C--BOUNDARY CONDITIONS
        DIMENSION HEAD(N).HEA(N)
C--INITIAL CONDITIONS AND ANSWERS
        DIMENSION R(N),R1(N)
C--THE GRID CHARACTERISTICS
        DIMENSION XLOC(N),YLOC(N),NOD(12.L).NODE(L),MAT(L)
C--LINKING INFORMATION
          DIMENSION YSPACE(20)  FLOWX(N),FLOWY(N)
C--MATRICIES USED FOR TEMPORARY STORAGE
         DIMENSION TPX(L),TPY(L),RI(N).G(N),NTIME(100),NSPACE(50)
C--INFORMATION FOR POINT SOURCES
          DIMENSION NFLUXW(Z,2).AFLUXW(Z),NFLUXH(Z.2),AFLUXH(Z)
C--INFORMATION FOR LINE SOURCES OF MASS OR WATER
      *.ALINEW(Z),ALINEH(Z),NLINEW(Z,2),NLINEH(Z,2)
C--INFORMATION FOR MASS BALANCE ON FLOW BOUNDARIES
          DIMENSION NWATER(Z).AWATER(Z),NHEAT(Z),AHEAT(Z)
C--ELEVEATION AT THE BOTTOM OF THE AQUIFER
          DIMENSION BOT(BBB)
C--INFORMATION ON CONVECTIVE BOUNDARIES
      * ,NCON(Z,3) TINF(Z),CONV(Z),ALOC(Z),NEL(Z,2) AEL(Z,2).CBAL(Z,2)
C--STRUCTURE MATRICIES
         DIMENSION GG(N), S(N,P),T(N,P)
       CALL URDATE(IDATE,IYEAR)
       CALL URTIMD(ITIME,ISEC)
1      FORMAT(1X,'MAXIMUM NUMBER OF NODES PERMITTED EXCEEDED')
3      FORMAT(1X,'MAXIMUM NUMBER OF ELEMENTS EXCEEDED')
4       FORMAT(1X,1H1)
       READ 7,(TITLE(J),J=1,24)
       PRINT 300 (TITLE(J),J=1,24),IDATE,IYEAR,ITIME,ISEC
7      FORMAT(12A6)
      READ,LM,LN,MBAND
       PRINT 8,LN,LM,MBAND
8      FORMAT(1X,'#OF NODES',I4 ' # OF ELEMENTS',I4 '  BANDWIDTH',I4)
       IF(LM.GT.L) PRINT 3
       IF(LN.GT.N) PRINT 1
      MPP=MBAND*2-1
       NAA=0
C**********************************************************
C    INPUT THE DATA
C
      CALL DATAIN(LM,LN MBAND.MAT,R1,R.HEA,HEAD,WX.WY,STO,INFLOW,PX,PY,
     *PCX,HEAT,XLOC,YLOC,NOD.TPX TPY,KRANA,CFACT,CFACT1,
     *ALPHA KSTEP,KBOUND.KFT,PCW,KRAN ALPH,KSTE,KBOUN,KF,PCH,DIFF.
     *Z,NCON,TINF,CONV,ALOC,LWATER NWATER,AWATER,LHEAT,NHEAT,AHEAT.
     *LFLUXW,NFLUXW,AFLUXW,LFLUXH,NFLUXH,AFLUXH,BBB,BOT.KTYPE
     *,NODE,NEL,AEL)
       CALL DI(KAREAL,ALPHM,ALPHAM,ER,ITER.LEL
     * LINEW,LINEH,NLINEW,NLINEH,ALINEW,ALINEH)
C
C**********************************************************
C  INITIALIZE THE STARTING ADDRESSES AND STARTING VALUES
C
       CALL FLOWS(LM,LN,R.R1,FLOWX,FLOWY)
C
       CALL BOUNDA(LM,LN.Z,R,R1,HEAD,HEA,FLOWX,FLOWY,HEAT,INFLOW
     * NWATER AWATER,NHEAT.AHEAT NCON.TINF
     * CONV,ALOC,NEL,AEL.CBAL,LEL,LHEAT,LWATER,PCW.AY,AZ
     *,LINEW,LINEH,NLINEW,NLINEH,ALINEW,ALINEH)
       CALL PARAM(LM,LN.BBB,XLOC,YLOC,NOD,PCX,DIFF,R,R1,WX,WY,TPX
     *,TPY,BOT.KAREAL)
       CALL VELOC(LN.LM,R1,WX,WY)
C
       MU=0
C***********************************************************
C  PROGRAM CONTROL IS ESTABLISHED
       READ,LA,LD,LE.LF,LG,LH
       IF(LE.GT.0) READ,(NTIME(I),I=1,LE)
       IF(LF.EQ.1) READ,ASIZE,NTHICK,MTHICK ERROR,XADD,YADD
       IF(LH GT.0) READ,(NSPACE(I),I=1,LH)
      IF(LA.EQ.1) CALL LAKE
       IF(LG.GT.0) CALL ADJUST(LG LM,LN Z,HEA,XLOC,YLOC,NOD
     *  ,FLOWX,FLOWY.R1)
        GO TO(10.20,30,30,30), KTYPE
10      CONTINUE
        MA=-1
        MB=1
        MC=1
        MD=1
        ME=1
       READ,MW
        GO TO 120
20      READ,MA,MB,MW,MC,MD
              IF(MD.LE.0) MU=1
        ME=1
        GO TO 120
```

```
30      READ,MO,MP.MV,MQ,MR.MS,MT MU,ME                                 MAI12100
        MA=-1                                                           MAI12200
        MB=1                                                            MAI12300
        MC=1                                                            MAI12400
        MD=1                                                            MAI12500
        ME=0                                                            MAI12600
       MW=9999999999                                                    MAI12700
        ALPH=0.0                                                        MAI12800
120      CONTINUE                                                       MAI12900
C   NA IS A COUNTER                                                     MAI13000
       NA=0                                                             MAI13100
C THESE LINES ESTABLISH NO CONVECTIVE BOUNDARIES IN EQN 1               MAI13200
       NCC=NCONV                                                        MAI13300
       NCONV=0                                                          MAI13400
C NO DISPERSION IN EQN 1                                                MAI13500
        KAD=0                                                           MAI13600
C   CONTROL FOR SYM OR ASSYM SOLUTION TECHNIQUE                         MAI13700
        NLB=NLA                                                         MAI13800
        NLA=LC                                                          MAI13900
       IF(KTYPE.EQ.5) GO TO 160                                         MAI14000
124    CONTINUE                                                         MAI14100
125    CONTINUE                                                         MAI14200
C                                                                       MAI14300
C***********************************************************            MAI14400
C                                                                       MAI14500
C     EQUATION NUMBER ONE                                               MAI14600
C                                                                       MAI14700
C***********************************************************            MAI14800
C ADJUST PERMEABILITIES FOR CHANGING TEMPERATURES                       MAI14900
       IF(MS.EQ.1)  CALL PE(MS)                                         MAI15000
C  MULTIPLICATION FACTOR FOR TIME STEP                                  MAI15100
       IF(KS.GT.0.OR.NA.GT.0) ALPH=ALPH*ALPHM                           MAI15200
C                                                                       MAI15300
C*******************************************                            MAI15400
C  INITALIZE STARTING ADDRESSES IN HEATER GAS AND BALANCE FOR EQUATION 1MAI15500
        MEQ=1                                                           MAI15600
      CALL LOAD(LM,LN,MBAND,NOD,NODE,R1,RI,G                            MAI15700
     *.GG,INFLOW,HEA.ALPH,KBOUN,KF,PCH,KRANA,CFACT1 KS,                 MAI15800
     *Z,NCON,TINF,CONV,ALOC,LWATER,NWATER,AWATER,CBAL.AY,AZ             MAI15900
     * ,AEL,NEL,MEQ,LEL,CBALA,LINEW,NLINEW,ALINEW)                      MAI16000
         CALL BALAN(LM,LN R1,WX,WY,STO HEA,INFLOW,Z,LWATER,AWATER       MAI16100
     *   ,LFLUXW,NFLUXW,WA,WB,WC,WD,WE,MPRINT,NTYPE,RI,NODE,NOD         MAI16200
     *FLOWX.FLOWY.PCW,MEQ,CBAL,CBALA,AY,AZ.NCON,KRANA)                  MAI16300
C                                                                       MAI16400
C                                                                       MAI16500
        CALL GAUSS(LLL.LM,LN.XLOC,YLOC,NOD,WY,WX,INFLOW,STO,ALPH.KBOUN, MAI16600
     * KRANA,Z,NCON,CONV,ALOC.NODE,CBAL,AY,AZ)                          MAI16700
      CALL STRUCT(LN,LM,MBAND,MPP,R1.S T,G GG,RI.HEA,XLOC,              MAI16800
     *YLOC,NOD,NODE,CFACT1,KRANA,ALPH,KBOUN)                            MAI16900
C                                                                       MAI17000
        NAA=NAA+1                                                       MAI17100
130     NA=NA+1                                                         MAI17200
        BKS=BKS+ALPH                                                    MAI17300
       IF(ALPH.LT.0.0001) BKS=AKS                                       MAI17400
        CALL HEATE(KS)                                                  MAI17500
       IF(KAREAL.EQ.1) CALL ITERAT(ER,ITER,KSA,$124)                    MAI17600
       KSA=0                                                            MAI17700
        IF(MC.GT.0) CALL MASBAL(ALPH)                                   MAI17800
        M1=NA/MD                                                        MAI17900
        MM1=M1*MD                                                       MAI18000
       IF(MM1.EQ.NA.AND.MU.EQ.0)  CALL WATER                            MAI18100
C                                                                       MAI18200
C                                                                       MAI18300
       IF(LG.GT.0) CALL ADJUS($125,$130)                                MAI18400
```

```
C
C
        M1=NA/MB
        M1=M1*MB
        IF(M1.EQ.NA) CALL WPRINT(BKS)
        M1=NA/MC
        M1=M1*MC
        IF(M1.EQ.NA) CALL BPRINT
C
C ADDS IN A CONSTANT BACKGROUND FLOW RATE
C
       IF(XADD.EQ.0.AND.YADD.EQ.0) GO TO 145
C
       CALL FLOADD
C
145    CONTINUE
C
C
        IF(MM1.EQ.NA.AND.MU.EQ.0) CALL WFLOW(BKS)
       M1=NA/MW
       M1=M1*MW
       IF(M1.EQ.NA) CALL FFLOW
       IF(MA.LE.BKS) GO TO 155
       IF(ALPHM.GT.1) GO TO 125
       GO TO 130
155    CONTINUE
        IF(ME.EQ.1) STOP
160    CONTINUE
C***********************************************************
C
C    EQUATION NUMBER TWO
C
C***********************************************************
       NTHICK=MTHICK
       NCONV=NCC
        KAD=MT
        NLA=NLB
C***************************************************************
C MULTIPLICATION FACTOR FOR THE TIME STEP
       IF(KS.GT.0) ALPHA=ALPHA*ALPHAM
C
C
C  INITIALIZE STARTING ADDRESSES IN HEATER AND BALANCE FOR EQUATION 2
       MEQ=2
C
      CALL LOAD(LM,LN,MBAND,NOD,NODE,R,RI,G,
     *GG,HEAT,HEAD,ALPHA,KBOUND,KFT,PCW,KRAN,CFACT,KS,
     *Z,NCON,TINF,CONV,ALOC,LHEAT,NHEAT,AHEAT,CBAL,AY,AZ
     * ,AEL,NEL,MEQ,LEL,CBALA,LINEH,NLINEH,ALINEH)
       CALL BALAN(LM,LN,R,PX,PY,PCX,HEAD,HEAT,Z,LHEAT,AHEAT,LFLUXH,
     *  NFLUXH,HA,HB,HC,HD,HE,MPRINT,NTYPE,RI,NODE,NOD,FLOWX,FLOWY,PCW,
     *  MEQ,CBAL,CBALA,AY,AZ,NCON,KRAN)
C
C
C
       CALL GAUSS(LLL,LM,LN,XLOC,YLOC,NOD,PY,PX,HEAT,PCX,ALPHA,KBOUND,KR
     *AN,Z,NCON,CONV,ALOC,NODE,CBAL,AY,AZ)
C
      CALL STRUCT(LN,LM,MBAND,MPP,R,S,T,G,GG,RI,HEAD,XLOC,YLOC,
     * NOD,NODE,CFACT,KRAN,ALPHA,KBOUND)
C
C
```

```
200     CONTINUE
      KS=KS+1
      AKS=AKS+ALPHA
C
C
C
C
        CALL HEATE(KS)
       IF(LH.EQ.0) GO TO 209
       DO 208 I=1.LH
       J=NSPACE(I)
 208   YSPACE(I)=R(J)
       WRITE(15,V) (YSPACE(I),I=1,LH)
209    CONTINUE
       IF(MQ.GT.0) CALL MASBAL(ALPHA)
       IF(ASIZE.NE.-1.0)        GO TO 212
      DO 211 I=2,LN 2
      R(I)=(R(I)+R(I-1))/2
211    R(I-1)=R(I)
212   CONTINUE
        M1=KS/MP
        M1=M1*MP
        IF(M1.EQ.KS) CALL HPRINT(AKS)
       M1=KS/MV
       M1=M1*MV
       IF(M1.EQ.KS) CALL FFFLOW
        M1=KS/MQ
        M1=M1*MQ
        IF(M1.EQ.KS) CALL BPRINT
        IF(MO.LE.AKS) STOP
C NTIME CONTAINS INFORMATION ON WHEN FLOWS ARE TO BE RECOMPUTED
C  NAA IS A COUNTER INDICATING THE NUMBER OF TIMES FLOWS HAVE BEEN COMPU
       IF(LE.EQ.1.AND.NTIME(NAA).EQ.KS) GO TO 120
        M1=KS/MR
        M1=MR*M1
       IF(M1.NE.KS) GO TO 200
       GO TO 120
C
300    FORMAT(1H1/1X,130('*')/1X,3('*',T130,'*'/1X),
     * 2('*',35X,12A6,T130,'*'/1X),'*',T130,'*'/1X,
     * 2('*',T100,A6,A2,T130.'*'/1X),130('*'))
C
C**************************************************************
C*****(********************************************************
C
       SUBROUTINE ITERAT(ER,ITER,KS.*)
       IF(KRANA.NE.0) RETURN
        A=0
       DO 1J=1,LN
       B=RI(J)-R1(J)
       B=ABS(B)
1      A=AMAX1(B,A)
       KS=KS+1
       IF(A.LT.ER.OR.KS.GT.ITER) PRINT 5,KS
       IF(A.LT.ER.OR.KS.GT.ITER) RETURN
       RETURN 4
5      FORMAT(1X//1X,'ITERATIONS NEEDED FOR CONVERGENCE  ',I5)
C
```

```
C*************************************************************
C
      SUBROUTINE FLOADD
C
       DO 140 KSS=1,LM
      CALL CORD(KSS)
      Y=YLOC(K4)-YLOC(K1)+YLOC(K3)-YLOC(K2)
      Y=ABS(Y)/2
      X=XLOC(K4)+XLOC(K1)-XLOC(K3)-XLOC(K2)
      X=ABS(X)/2
      XX=1.0
      YY=1.0
      IF(NTHICK.EQ.0) GO TO 135
      YY=(YLOC(K1)+YLOC(K4))/2*ASIZE
      XX=(XLOC(K3)+XLOC(K4))/2*ASIZE
      XX=ABS(XX)
      YY=ABS(YY)
135   CONTINUE
      FLOWY(KSS)=FLOWY(KSS)+YADD*X/YY
      FLOWX(KSS)=FLOWX(KSS)+XADD*Y/XX
140   CONTINUE
      END
```

```
C**************************************************************
C
C    THIS ROUTINE IS USED TO READ IN THE DATA
C
      SUBROUTINE DATAIN(LM,LN,MBAND,MAT,R1,R,HEA,HEAD,WX,WY,STO
     *,INFLOW,PX,PY,PCX,HEAT,XLOC,YLOC,NOD,TPX,TPY,KRANA,CFACT,CFACT1,
     *ALPHA,KSTEP,BOUND,KFT,PCW,KRAN,ALPH,KSTE,BOUN,KF,PCH,DIFF,
     *Z,NCON,TINF,CONV,ALOC,LWATER,NWATER,AWATER,LHEAT,NHEAT,AHEAT,
     ,LFLUXW,NFLUXW,AFLUXW,LFLUXH,NFLUXH,AFLUXH,BBB,BOT,KTYPE
     *,NODE,NEL,AEL)
       INTEGER BOUND,BOUN,BBB,Z
      REAL INFLOW,MAT
      COMMON/HI/TITLE(25),V(26),VV(26)/HH/LFLOW,LON
      COMMON/A1/M,MM,NUMNP,NSIZE/AM/NLA,PCT
       COMMON/CON/MC1,MC2,NCONV
       DIMENSION NCON(Z,3),TINF(Z),CONV(Z),ALOC(Z),NWATER(Z),
     * AWATER(Z),NHEAT(Z),AHEAT(Z),NFLUXW(Z,2),AFLUXW(Z),NFLUXH(Z,2),
     *AFLUXH(Z),BOT(BBB),AEL(Z,2),NEL(Z,2)
      DIMENSION MAT(LM),R1(LN),R(LN),HEA(LN),HEAD(LN),WX(LM),WY(LM)
      DIMENSION STO(LM),INFLOW(LM),PX(LM),PY(LM),PCX(LM),HEAT(LM)
      DIMENSION XLOC(LN),YLOC(LN),NOD(12,LM), TPX(LM),TPY(LM)
       DIMENSION FF(8),Q(12), A(8,50),DIFF(LM,2),NODE(LM)
       RETURN
C
C
       ENTRY DI(KAREAL,ALPHM,ALPHAM,ER,ITER,LEL,
     *LINEW,LINEH,NLINEW,NLINEH,ALINEW,ALINEH)
       DIMENSION NLINEW(Z,2),NLINEH(Z,2),ALINEW(Z),ALINEH(Z)
C***********************************************************
C
      DO 2 J=1,LM
      HEAD(J)=0.0
 2    HEA(J)=0.0
C
C
C***********************************************************
C
    7 FORMAT(12A6)
        READ,KTYPE,KPRINT,LON,MQ,NLA,CFACT
       CFACT1=CFACT
        NUMNP=LN
      NSIZE=NUMNP
        M=LM
     MM=M
      READ 40,(V(I),I=1,26)
      READ 40,(VV(I),I=1,26)
       IF(NLA.GT.0) PRINT 155,NLA
C
       READ 7,(Q(J),J=1,12)
C*************************************************************
C
C         DATA GROUP II
C     READ IN DATA ON THE SPATIAL STRUCTURE
C
      IF(MQ.LT.0.0001) GO TO 12
      DO 8 I=1,NUMNP
      READ,J,XLOC(J),YLOC(J)
    8 CONTINUE
      DO 9 II=1,LM
       CALL ELREAD
    9 CONTINUE
      GO TO 15
```

```
12      CONTINUE
      READ,KSPACX,KSPACY
       CALL P(KSPACX,KSPACY)
       MQ=ABS(MQ)
       DO 14 J=1,LM
14     NODE(J)=4
   15  CONTINUE
C
       READ 7,(Q(J),J=1,12)
C*************************************************************
C
C       DATA GROUP III
C   INPUT INITIAL CONDITIONS AND BOUNDARY CONDITIONS
C
      KRAN=0
      KRANA=0
        GO TO (21,20,23,22,22),KTYPE
20     READ,ALPH,ALPHM,BOUN
       PCH=1.0
       KF=0
        PRINT 150,ALPH
      KRANA=1
       CALL READER(LN,R1,I)
       IF(I.NE.999) STOP R1
21     CALL READER(LN,HEA,I)
       IF(I.NE.999) STOP HEA
      GO TO 24
22     READ,ALPH,ALPHM,BOUN
       KF=0
       PCH=1.0
        PRINT 150,ALPH
      KRANA=1
       CALL READER(LN,R1,I)
       IF(I.NE.999) STOP R1
23     READ,ALPHA,ALPHAM,BOUND,PCW
       KFT=0
       IF(NLA.EQ.0) KFT=1
        PCT=PCW
        PRINT 151,ALPHA,PCW
      KRAN=1
       CALL READER(LN,R,I)
       IF(I.NE.999) STOP R
       CALL READER(LN,HEA,I)
        IF(I.NE.999) STOP HEA
       CALL READER(LN,HEAD,I)
       IF(I.NE.999) STOP HEAD
24    CONTINUE
       IF(KFT.EQ.1) PRINT 156
C
       READ 7,(Q(J),J=1,12)
C***********************************************************
C
C       DATA GROUP IV
C   INPUT PARAMETERS
C
      READ,  NMATA
      IF(KTYPE.GT.2) READ,(FF(I),I=1,8)
C
       IF(KTYPE.LT.3) READ,FF(1),FF(2),FF(3)
      PRINT 105,(FF(I),I=1,8)
       PRINT 102
      CALL READER(LM,MAT,I)
      IF(I.NE.999) STOP MAT
      DO 32 JJ=1,NMATA
        IF(KTYPE.LT.3) READ,J,(A(I,J),I=1,3)
       IF(KTYPE.EQ.5) READ,J,(A(I,J),I=4,8)
      IF(KTYPE.GT.2) READ,J,(A(I,J),I=1,8)
       DO 31 I=1,8
31     A(I,J)=A(I,J)*FF(I)
      PRINT 104,J,(A(I,J),I=1,8)
32    CONTINUE
      DO 34 K=1,LM
      DO 33 J=1,50
       KZX=MAT(K)
      IF(KZX.NE.J) GO TO 33
      WX(K)=A(1,J)
      WY(K)=A(2,J)
      STO(K)=A(3,J)
      PX(K)=A(4,J)
      PY(K)=A(5,J)
      PCX(K)=A(6,J)
      DIFF(K,1)=A(7,J)
      DIFF(K,2)=A(8,J)
      GO TO 34
33    CONTINUE
 34    CONTINUE
      DO 35 J=1,LM
      TPX(J)=WX(J)
35    TPY(J)=WY(J)
C
        PRINT 120
      PRINT VV,(MAT(J),J=1,LM)
C
       READ 7,(Q(J),J=1,12)
C*****************************************************************
C
C   INPUT DISTRIBUTED SOURCES
C
        READ,KGEN,KGENH
       IF(KGEN.EQ.1) CALL READER(LM,INFLOW,I)
       IF(I.NE.999) STOP INFL
       IF(KGENH.EQ.1) CALL READER(LM,HEAT,I)
       IF(I.NE.999) STOP HEAT
C***************************************************************
C
C  INPUT POINT SOURCES OF WATER AND HEAT
      READ,LWATER,LHEAT
      IF(LWATER.EQ.0) GO TO 61
      READ,(NWATER(I),AWATER(I),I=1,LWATER)
      PRINT 131,(NWATER(I),AWATER(I),I=1,LWATER)
61    CONTINUE
      IF(LHEAT.EQ.0) GO TO 63
      READ,(NHEAT(I),AHEAT(I),I=1,LHEAT)
      PRINT 132,(NHEAT(I),AHEAT(I),I=1,LHEAT)
63    CONTINUE
C
C
C INPUT LINE SOURCES OF WATER AND HEAT
       READ,LINEW,LINEH
       IF(LINEW.EQ.0) GO TO 66
       READ,(NLINEW(I,1),NLINEW(I,2),I=1,LINEW)
       READ,(ALINEW(I),I=1,LINEW)
       PRINT 278,(NLINEW(I,1),NLINEW(I,2),ALINEW(I),I=1,LINEW)
66     IF(LINEH.EQ.0) GO TO 68
       READ,(NLINEH(I,1),NLINEH(I,2),I=1,LINEH)
       READ,(ALINEH(I),I=1,LINEH)
       PRINT 279,(NLINEH(I,1),NLINEH(I,2),ALINEH(I),I=1,LINEH)
```

```
68     CONTINUE                                                         DAT19000
C                                                                       DAT19100
C                                                                       DAT19200
       READ 7,(Q(J),J=1,12)                                             DAT19300
C************************************************************           DAT19400
C                                                                       DAT19500
C        DATA GROUP VI                                                  DAT19600
C                                                                       DAT19700
C INPUT INFORMATION NEEDED FOR A 2-D AREAL VIEW OF THE SYSTEM           DAT19800
       READ,KAREAL                                                      DAT19900
       IF(KAREAL.EQ.0) GO TO 41                                         DAT20000
       READ,ER,ITER                                                     DAT20100
       CALL READER(LN,BOT,I)                                            DAT20200
       IF(I.NE.999)  STOP BOT                                           DAT20300
       CALL READER(LN,R1,I)                                             DAT20400
       IF(I.NE.999) STOP R1 2                                           DAT20500
40     FORMAT(2(13A6,A2/))                                              DAT20600
       PRINT 276                                                        DAT20700
       PRINT V,(BOT(I),I=1,LN)                                          DAT20800
C                                                                       DAT20900
41    CONTINUE                                                          DAT21000
       READ 7,(Q(J),J=1,12)                                             DAT21100
C************************************************************           DAT21200
C INPUT CONVECTIVE BOUNDARY INFORMATION                                 DAT21300
C                                                                       DAT21400
C         DATA GROUP VII                                                DAT21500
       IF(KTYPE.LT.3) GO TO 59                                          DAT21600
        READ,NCONV                                                      DAT21700
      IF(NCONV.EQ.0) GO TO 55                                           DAT21800
       READ,(NCON(I,1),NCON(I,2),CONV(I),I=1,NCONV)                     DAT21900
C                                                                       DAT22000
      READ,(TINF(I),I=1,NCONV)                                          DAT22100
 55   CONTINUE                                                          DAT22200
C                                                                       DAT22300
C                                                                       DAT22400
C INFORMATION NEEDED FOR HEAT OR MASS_TRANSFER ACROSS A BOUNDARY        DAT22500
C  WITH A SPECIFIED HEAD                                                DAT22600
C                                                                       DAT22700
       READ,LEL                                                         DAT22800
       IF(LEL.EQ.0) GO TO 56                                            DAT22900
       READ,(NEL(J,1),NEL(J,2),J=1,LEL)                                 DAT23000
       READ,(AEL(J,2),J=1,LEL)                                          DAT23100
       PRINT 157,(NEL(J,1),NEL(J,2),AEL(J,2),J=1,LEL)                   DAT23200
56     CONTINUE                                                         DAT23300
        PRINT 110,(NCON(I,1),NCON(I,2),CONV(I),I=1,NCONV)               DAT23400
59     CONTINUE                                                         DAT23500
       READ 7,(Q(I),I=1,12)                                             DAT23600
C                                                                       DAT23700
C************************************************************           DAT23800
C                                                                       DAT23900
C       DATA GROUP VIII                                                 DAT24000
C   INPUT LOCATION OF FLOW BOUNDARIES                                   DAT24100
C       INPUT NODE # AND THEN THE BOUNDARY CODE                         DAT24200
      READ,LFLUXW,LFLUXH                                                DAT24300
      IF(LFLUXW.EQ.0) GO TO 71                                          DAT24400
      READ,(NFLUXW(I,1),NFLUXW(I,2),I=1,LFLUXW)                         DAT24500
      PRINT 133,(NFLUXW(I,1),NFLUXW(I,2),I=1,LFLUXW)                    DAT24600
71    CONTINUE                                                          DAT24700
      IF(LFLUXH.EQ.0) GO TO 73                                          DAT24800
      READ,        (NFLUXH(I,1),NFLUXH(I,2),I=1,LFLUXH)                 DAT24900
      PRINT 134,   (NFLUXH(I,1),NFLUXH(I,2),I=1,LFLUXH)                 DAT25000
73     CONTINUE                                                         DAT25100
C                                                                       DAT25200
       READ 7,(Q(I),J=1,12)                                             DAT25300
```

```
C
C FORMAT INFORMATION--INFORMATION PRINTED ON EVERY RUN
102    FORMAT(1X//1X,T10,'MATERIAL',T20,'HOR PERM',T32,' VER PERM',T50,
     *'STORAGE',T65,'HOR THERM COND',T85,'VERT THERM COND',T103,
     *'SPECIFIC  DISPERSION COEFS'/1X,T103,'HEAT')
104   FORMAT(1X,T12,G9.4,T22,G9.4,T34,G9.4,T50,G9.4,T68,G9.4,T88,
     *G9.4,T105,G9.4,T115,2G9.4/)
105     FORMAT(1X//1X,'PARAMETER FACTORS--IN ORDER     ',8(G10.6,1X))
110    FORMAT(1X,'CONVECTIVE OUT AND CONDUCTIVE BOUNDARY INFORMATION',1X
     *,'          ELEM #--BOUNDARY CODE--TRANSFER COEFFICIENT'
     * /1X,20(1X,5(I4,1X,I1,1X,G11.6,2X)/))
120   FORMAT(1X//1X,'THE VALUES IN THE TYPE OF MATERIAL MATRIX ARE')
131   FORMAT(1X//1X,'POINT SOURCES OF WATER--NODE # AND AMOUNT'/1X,
     *5(I4,1X,G12.6,5X))
132   FORMAT(1X//1X,'POINT SOURCES OF HEAT'/1X,5(I4,1X,G12.6,5X))
133   FORMAT(1X//1X,'WATER FLOW BOUNDARIES--ELEM # AND BOUNDARY CODE'/
     *10(I4,2X,I1,5X))
134   FORMAT(1X//1X,'HEAT FLOW BOUNDARIES--NODE # AND BOUNDARY CODE'/
     *10(I4,2X,I1,5X))
150   FORMAT(1X//1X,'EQUATION 1 TIME STEP',G12.6)
151   FORMAT(1X//1X,'EQUATION 2 TIME STEP',G12.6,10X,'HEAT OR MASS CAPAC
     *ITY COEFFICIENT',G12.6)
155    FORMAT(1X//1X,'VELOCITIES ARE BEING CALCULATED AT CURRENT TIME ST
     *EP    ',I1)
156    FORMAT(1X//1X,'VELOCITIES ARE BEING CALCULATED AT THE PREVIOUS TI
     *ME STEP')
157    FORMAT(1X//1X,'INFORMATION FOR MASS OR HEAT TRANSFER ACROSS'
     *,1X,'A SPECIFIED HEAD BOUNDARY'/1X,'ELEMENT NUMBER--BOUNDARY CODE'
     *,1X,'--TEMPERATURE OR CONCENTRATION OF INCOMING FLUID'
     */1X,5(2X,I4,1X,I2,1X,2X,G10.5)/)
C*********************************************************************
C          THE ROUTINE THAT PRINTS OUT THE INITIAL CONDITIONS
C            SPECIFIED BY THE INPUT DATA
C
      IF(KPRINT.EQ.0) GO TO 300
      PRINT 199,(TITLE(J),J=1,24)
  199 FORMAT(1H1,1X//,20('*',12A6,20('*')/1X,10X,'THE UNITS USED ARE',
     *12A6))
      PRINT 210,NUMNP,M
  210  FORMAT(1X,'NUMBER OF NODES ',I3,'   NUMBER OF ELEMENTS ',I3//)
      PRINT 211,ALPHA,PCW
  211 FORMAT(1X,10X,'TIME STEP',F5.2,' UNITS',10X,'PC=',G10.5)
      PRINT 220
  220 FORMAT(1X,'ELEMENT DATA'/1X,'NODE NUMBER',T20,'X-LOCATION',T40,
     *'Y-LOCATION',T60,'SPEC. HEAD',T75,'SPEC. T',T90,'INITIAL T',
     * T110,'INITIAL HEAD')
      DO 240 J=1,NUMNP
      PRINT 225,J,XLOC(J),YLOC(J),HEA(J),HEAD(J),R(J),R1(J)
225   FORMAT(1X,T5,I3,T20,G10.5,T40,G10.5,T60,G10.5,T75,G10.5,T90,G10.5,
     *  T110,G10.5)
  240 CONTINUE
      PRINT 250
250    FORMAT(1X,2X///1X,'ELEMENT PROPERTIES'/1X, 'ELEMENT',T12,'HCX',
     *T22,'HCY',T32,'STO',T42,'TCX',T52,'TCY',T63,'PC',T72,
     *  ' THE ELEMENT NODES    ',T102,'HEATIN',T112,'WATER IN')
      DO 270 J=1,M
      PRINT 275,J,WX(J),WY(J),STO(J),PX(J),PY(J),PCX(J),(NOD(LZ,J),LZ=1,
     *    4),HEAT(J),INFLOW(J)
275   FORMAT(1X,T4,I3,T10,G8.3,T20,G8.3,T30,G8.3,T40,G8.3,,T50,G8.3,
     *  T60,G8.3,T70,4I3,T100,G8.3,T110,G8.3)
276    FORMAT(1X//1X,'ELEVATION OF THE AQUIFER BOTTOM',1X/)
278    FORMAT(1X,'LINE SOURCES OF WATER--ELEMENT NUMBER,BOUNDARY CODE,RA
     *TE'/1X,6(I4,1X,I2,1X,G10.5,2X))
```

```
279    FORMAT(1X,'LINE SOURCES OF HEAT--ELEMENT NUMBER,BOUNDARY CODE, RADAT31700
     *TE'/1X,6(I4,1X,I2,1X,G10.5,2X))                                   DAT31800
  270 CONTINUE                                                          DAT31900
300    CONTINUE                                                         DAT32000
C ROUTINE TO ORIENTATE THE MATRIX--INSURES THAT ALL PROBLEMS ARE ORIENTADAT32100
C    IN THE SAME MANNER                                                 DAT32200
C                                                                       DAT32300
       IF(MQ.EQ.0) GO TO 294                                            DAT32400
       IF(MQ.EQ.2) GO TO 294                                            DAT32500
       AC=0                                                             DAT32600
       AB=0                                                             DAT32700
       DO 282 J=1,LN                                                    DAT32800
       AB=AMAX1(XLOC(J),AB)                                             DAT32900
282    AC=AMAX1(YLOC(J),AC)                                             DAT33000
       IF(MQ.EQ.4) GO TO 288                                            DAT33100
       DO 284 J=1,LN                                                    DAT33200
284    YLOC(J)=AC-YLOC(J)                                               DAT33300
       IF(MQ.EQ.1) GO TO 294                                            DAT33400
288    CONTINUE                                                         DAT33500
       DO 290 J=1,LN                                                    DAT33600
290    XLOC(J)=AB-XLOC(J)                                               DAT33700
294    CONTINUE                                                         DAT33800
C                                                                       DAT33900
C***********************************************************************DAT34000
      RETURN                                                            DAT34100
      SUBROUTINE ELREAD                                                 DAT34200
       DIMENSION DIGIT(10), CHAR(80)                                    DAT34300
       DATA DIGIT/'0','1','2','3','4','5','6','7','8','9'/              DAT34400
54     FORMAT(1X,'PROBLEMS--BAD CHARACTER',1X,A1,3X,'ELEMENT',I6)       DAT34500
55     FORMAT(1X,'PROBLEM--NOT ENOUGH NODAL DATA FOR ELEMENT    ',I6)   DAT34600
       READ 1,(CHAR(J),J=1,80)                                          DAT34700
1      FORMAT(80A1)                                                     DAT34800
       KC=0                                                             DAT34900
       KE=4                                                             DAT35000
       KS=2                                                             DAT35100
       J=0                                                              DAT35200
       KL=0                                                             DAT35300
8      NUM=0                                                            DAT35400
9      J=J+1                                                            DAT35500
       IF(CHAR(J).EQ.' ') GO TO 9                                       DAT35600
10     I=0                                                              DAT35700
11     I=I+1                                                            DAT35800
       IF(I.LE.10) GO TO 12                                             DAT35900
       PRINT 54,CHAR(J),L                                               DAT36000
       STOP                                                             DAT36100
12     IF(CHAR(J).NE.DIGIT(I)) GO TO 11                                 DAT36200
       NUM=10*NUM+I-1                                                   DAT36300
       J=J+1                                                            DAT36400
       IF(CHAR(J).EQ.' ') GO TO 13                                      DAT36500
       IF(CHAR(J).EQ.'*') GO TO 14                                      DAT36600
        GO TO 10                                                        DAT36700
13     KL=KL+1                                                          DAT36800
       IF(KL.EQ.1) L=NUM                                                DAT36900
       IF(KL.EQ.1) GO TO 8                                              DAT37000
       KL=KL+1                                                          DAT37100
       KC=KC+1                                                          DAT37200
       IF(KC.EQ.5) GO TO 15                                             DAT37300
       NOD(KC,L)=NUM                                                    DAT37400
       IF(KS.EQ.0) KE=KE+2                                              DAT37500
       IF(KS.EQ.1) KE=KE+1                                              DAT37600
       KS=0                                                             DAT37700
       GO TO 8                                                          DAT37800
14     KE=KE+1                                                          DAT37900
       KS=KS+1                                                          DAT38000
      NOD(KE,L)=NUM
      IF(J.LT.65) GO TO 8
      PRINT 55,L
      STOP
15    MTM=4
      DO 16 I=5,12
      IF(NOD(I,L).EQ.0) GO TO 16
      MTM=MTM+1
16    CONTINUE
      NODE(L)=MTM
      RETURN
C
C**************************************************************
C
      SUBROUTINE P(KX,KY)
      DIMENSION D(50),  B(50),C(50)
     J=0
      MN=0
      IF(KX.LT.0) MN=1
     KX=ABS(KX)
     KXA=KX-1
     KYA=KY-1
     DO 2 L=1,KXA
     DO 1 K=1,KYA
     J=J+1
     NOD(4,J)=L-1+J
     NOD(3,J)=L+J+KYA
     NOD(1,J)=L+J
     NOD(2,J)=L+J+KY
 1   CONTINUE
 2   CONTINUE
      READ,(B(J),J=1,KX)
       PRINT 10
      PRINT  ,(B(J),J=1,KX)
     READ,(C(J),J=1,KY)
      PRINT 11
      PRINT,(C(J),J=1,KY)
     J=0
     DO 4  L=1,KX
     DO 3 K=1,KY
     J=J+1
     YLOC(J)=C(K)
     XLOC(J)=B(L)
 3   CONTINUE
 4   CONTINUE
      IF(MN.EQ.0) RETURN
      READ,MLD
      MLB=1
      IF(MLD.EQ.0) MLB=KY
      READ, (D(J),J=1,KX)
      KXX=LN-KY+1
      DO 20 K=MLB,KY
      L=0
      DO 20 JJ=1,KXX,KY
      J=JJ+K-1
      L=L+1
20    YLOC(J)=YLOC(J)+D(L)
     RETURN
10    FORMAT(1X/1X,'THE X-SPACING IS')
11    FORMAT(1X/1X,'THE Y-SPACING IS')
      RETURN
      SUBROUTINE READER(LZ,Q,I)
      DIMENSION Q(LZ)                                                   DAT44300
      READ,QI,IZ,FACT                                                   DAT44400
      DO 1 IP=1,LZ                                                      DAT44500
    1 Q(IP)=QI                                                          DAT44600
      IF(IZ.GE.0) GO TO 2                                               DAT44700
      READ,(Q(IP),IP=1,LZ),I                                            DAT44800
      GO TO 3                                                           DAT44900
    2 I=999                                                             DAT45000
      IF(IZ.EQ.0) GO TO 3                                               DAT45100
      READ,(J,Q(J),IP=1,IZ)                                             DAT45200
    3 CONTINUE                                                          DAT45300
      DO 5 IP=1,LZ                                                      DAT45400
    5 Q(IP)=Q(IP)*FACT                                                  DAT45500
      RETURN                                                            DAT45600
      END                                                               DAT45700
```

```
C***********************************************************************
C
C     THIS ROUTINE ASSEMBLES THE STRUCTURE MATRICIES AND PREFORMS
C        A FORWARD REDUCTION OF THE STIFNESS MATRIX BY GAUSS ELIMINATION
C
      SUBROUTINE STRUCT(LN,LM,MBAND,MP,R,S,T,G,GG,RI,HEAD,XLOC,
     *YLOC,NOD,NODE,CFACT,TT,ALPHA,KBOUND)
       INTEGER TT
       REAL JAC
      DOUBLE PRECISION AA,SE,TE,S,T
      COMMON/AAA/XL(4),YL(4),DETJAC,JAC(4,4)/A1/M,MM,NUMNP,NSIZE
       COMMON/AB/AA(6),W(4)/ABB/SE(12,12),TE(12,12)
       COMMON/AC/RE(12),GE(12)/ACC/KK(12),K1,K2,K3,K4
       COMMON/HH/LFLOW,LON/HM/PXX,PYY,PXY,KAD/AM/KSYM,PCT
      DIMENSION XLOC(LN),YLOC(LN),NOD(12,LM),R(LN).S(LN,MP)
      DIMENSION T(LN,MP),G(LN),GG(LN),RI(LN),NODE(LM),HEAD(LN)
      LFLOW=1
      AA(1)=-(1/3)**.5
      AA(2)=-AA(1)
C
       DO 50 J=1,LN
       GG(J)=0.0
        DO 50 I=1,MP
       S(J,I)=0.0
50     T(J,I)=0.0
C PRINT MESSAGE IF DISPERSION ROUTINE IS USED
       IF(KAD.EQ.1) PRINT 10
C
C***********************************************************************
C
C      THE STIFFNESS MATRIX FORMULATION ROUTINE
C
       DO 150 LLL=1,MM
      PXX=0
      PYY=0
      PXY=0
       M4=NODE(LLL)
       CALL DERIVE(LLL,M4)
C***********************************************************************
C          THE STRUCTURE STIFFNESS MATRIX ASSEMBLER
       K=0
       DO 110 I=1,M4
100      K=K+1
       IF(NOD(K,LLL).EQ.0) GO TO 100
110    KK(I)=NOD(K LLL)
      DO 140 I=1,M4
      IF(KK(I).LE.0) GO TO 140
      II=KK(I)
C
       GG(II)=GG(II)+GE(I)
120    CONTINUE
      DO 140 J=1,M4
           IF(KSYM.GT.0) GO TO 130
      IF(KK(J).LT.II) GO TO 140
      JJ=KK(J)-II+1
      GO TO 135
130       CONTINUE
          JJ=KK(J)-II+MBAND
135     CONTINUE
       IF(JJ.GT.MP) PRINT 20,LLL.JJ
       S(II,JJ)=S(II,JJ)+SE(I,J)
      IF(TT.EQ.0) GO TO 140
      T(II,JJ)=T(II,JJ)+TE(I,J)
```

```
  140 CONTINUE
  150  CONTINUE
C
C
C*****************************************************************************
C
C--ROUTINE FOR SPECIFIED HEAD BOUNDARY CONDITIONS
      DO 200 N=1,NUMNP
      IF(HEAD(N).EQ.0) GO TO 200
      IF(KSYM.EQ.0) S(N,1)=S(N,1)+CFACT
      IF(KSYM.GT.0) S(N,MBAND)=S(N,MBAND)+CFACT
200     CONTINUE
C
C*****************************************************************************
C
C   CALL THE ROUTINE THAT CHECKS FOR THE STABILITY OF THE CHOSEN TIME ST
       KSM=KSYM
      IF(LON.EQ.99.AND.TT.EQ.1) CALL EIGEN(LN,MBAND,MP,KSM,CFACT,S,T,RI,
     *HEAD)
C GAUSS ELIMINATION--FORWARD REDUCTION OF STIFNESS MATRIX
C
        IF(KSYM.EQ.0) CALL SOLVE(LN MBAND,S,T,R,RI)
        IF(KSYM.NE.0) CALL ASOLVE(S,R,RI,LN,MBAND-1,LN,MBAND*2-1,T)
C
C
10     FORMAT(1X/1X,'THE DISPERSION ROUTINE IS BEING USED')
20     FORMAT(1X/1X,'MAX. BANDWIDTH EXCEEDED IN ELEMENT',I4,
     *  'WHERE BANDWIDTH IS  ',I4)
      RETURN
C
C
       END
```

```
C*********************************************************************** GAU  100
C*                                                                      GAU  200
C   THIS ROUTINE FORMS ELEMENT STIFFNESS MATRIX BY GAUSS QUADRATURE     GAU  300
C                                                                       GAU  400
      SUBROUTINE GAUSS(LLL,LM,LN,XLOC,YLOC,NOD,PY,PX,HEAT,PCX,ALPHA,KBOGAU  500
     *UND,TT,Z,NCON,CONV,ALOC,NODE,CBAL,AY,AZ)                          GAU  600
      REAL JAC                                                          GAU  700
      INTEGER VE, TT,Z,AY(4),AZ(4)                                      GAU  800
       DOUBLE PRECISION AA,NOT,NXI,NET,SE,TE,BB ,DUM1,DUM2,DUM3         GAU  900
       COMMON/AA/NXI(12),NET(12),NOT(12)                                GAU 1000
       COMMON/AAA/XL(4),YL(4),DETJAC,JAC(4,4)                           GAU 1100
       COMMON/AB/AA(6),W(4)/ABB/SE(12,12),TE(12,12)                     GAU 1200
       COMMON/AC/RE(12),GE(12)/ACC/KK(12),K1,K2,K3,K4                   GAU 1300
      COMMON/HH/LFLOW,LON/HM/PXX,PYY,PXY,KAD/ME/MEQ                     GAU 1400
      COMMON/CON/MC1,MC2,NCONV/AS/BB(2,12)                              GAU 1500
               DIMENSION NCON(Z,3),CONV(Z),ALOC(Z)                      GAU 1600
      COMMON/CON1/HE(4),HEE(4,4)/AM/NLA,PCH/AT/XII(4),ETI(4)            GAU 1700
       COMMON/ATHICK/ASIZE,NTHICK,THICK,ERROR                           GAU 1800
       DIMENSION VE(4),AVY(4),AVX(4),VEL(4)                             GAU 1900
      DIMENSION            XLOC(LN),YLOC(LN),PY(LM),PX(LM),HEAT(LM),PCX(LGAU 2000
     *M),CBAL(Z,2),AG(6),NODE(LM),NOD(12,LM)                            GAU 2100
       DATA XII/-1.,1.,1.,-1./                                          GAU 2200
       DATA VE/1,4,2,3/                                                 GAU 2300
       DATA ETI/-1.,-1.,1.,1./                                          GAU 2400
       DATA W/.3478549,.6521451,.6521451,.3478549/                      GAU 2500
       DATA AG/-.5773503,.5773503,-.8611363,-.3399810,.3399810,.8611363/GAU 2600
      RETURN                                                            GAU 2700
C***********************************************************************GAU 2800
C                                                                       GAU 2900
       ENTRY DERIVE(LLL,M4)                                             GAU 3000
C                                                                       GAU 3100
C                                                                       GAU 3200
C*****************************************************************      GAU 3300
C                                                                       GAU 3400
        CALL CORD(LLL)                                                  GAU 3500
       T=1.0                                                            GAU 3600
       CALL PEE(T,LLL)                                                  GAU 3700
      DO 10 K=1,12                                                      GAU 3800
      GE(K)=0.0                                                         GAU 3900
      DO 10 L=1,12                                                      GAU 4000
      TE(K,L)=0.0                                                       GAU 4100
   10 SE(K,L)=0                                                         GAU 4200
C     START THE QUADRATURE LOOP                                         GAU 4300
       NP=2                                                             GAU 4400
       IF(M4.GT.4) NP=4                                                 GAU 4500
       KVE=0                                                            GAU 4600
        DO 200 II=1,NP                                                  GAU 4700
       DO 200 JJ=1,NP                                                   GAU 4800
       KVE=KVE+1                                                        GAU 4900
       IF(NP.EQ.4) GO TO 20                                             GAU 5000
       XI=AG(JJ)                                                        GAU 5100
       YI=AG(II)                                                        GAU 5200
       GO TO 30                                                         GAU 5300
20     XI=AG(JJ+2)                                                      GAU 5400
       YI=AG(II+2)                                                      GAU 5500
       IF(LFLOW.EQ.0) XI=0.0                                            GAU 5600
       IF(LFLOW.EQ.0) YI=0.0                                            GAU 5700
30     CONTINUE                                                         GAU 5800
       IF(M4.GT.4) CALL SHAPE(LLL,M4,XI,YI,XLOC,YLOC,LN,LM,NOD)         GAU 5900
        IF(M4.EQ.4) CALL SHAPE1(II,JJ)                                  GAU 6000
C                                                                       GAU 6100
C********************************************                           GAU 6200
```

```
175   CONTINUE
      THICK=1.0
      IF(NTHICK.EQ.0) GO TO 176
      XX=(XLOC(K2)+XLOC(K1))/2
      XX=ABS(XX)
      THICK=XX*ASIZE
     YY=(YLOC(K3)+YLOC(K1))/2
     YY=ABS(YY)
     IF(ASIZE.LT.0) THICK=-YY*ASIZE
176     DUM1=W(II)*W(JJ)*DETJAC*THICK*T
      IF(M4.EQ.4) DUM1=DETJAC*THICK*T
C*******************************************
C
       IF(LFLOW.EQ.0) DETJAC=DETJAC*T*THICK
       IF(LFLOW.EQ.0) RETURN
C VELEOCITY AND DISPERSION CALCULATIONS
C
      VX=0.0
      VY=0.0
       IF(NLA.EQ.0) GO TO 178
        IF(NLA.EQ.1) CALL VELO(LLL,VX,VY)
        IF(NLA.EQ.2) CALL VCENT(LLL,VX,VY)
       IF(KAD.EQ.1) CALL MECD(LLL,VX,VY)
C
       IF(KVE.GT.4) GO TO 178
       KEE=VE(KVE)
       AVY(KEE)=VY
       AVX(KEE)=VX
178    CONTINUE
C
C*******************************************
C
C
C*******************************************
C
      DO 200 NROW=1,M4
     GE(NROW)=GE(NROW)+NOT(NROW)*HEAT(LLL)*DUM1
      DO 200 NCOL=1,M4
       IF(NLA.EQ.0) GO TO 180
       DUM2=BB(1,NROW)*NOT(NCOL)*PCH*VX+BB(2,NROW)*NOT(NCOL)*PCH*VY
       SE(NROW,NCOL)=SE(NROW,NCOL)+DUM2*DUM1
180     CONTINUE
      DUM2=BB(1,NROW)*BB(1,NCOL)*(PX(LLL)+PXX)
      SE(NROW,NCOL)=SE(NROW,NCOL)+DUM1*DUM2
      DUM2=BB(2,NROW)*BB(2,NCOL)*(PY(LLL)+PYY)
      SE(NROW,NCOL)=SE(NROW,NCOL)+DUM1*DUM2
C     THESE NEST TWO LINES ADD IN TRANSVERSE TERMS OF CONDUCTIVITY TENSO
       DUM2=BB(1,NROW)*BB(2,NCOL)*PXY+BB(2,NROW)*BB(1,NCOL)*PXY
     SE(NROW,NCOL)=SE(NROW,NCOL)+DUM1*DUM2
      IF(TT.EQ.0) GO TO 200
      DUM3=NOT(NROW)*NOT(NCOL)*PCX(LLL)
       IF(NLA.EQ.0) DUM3=DUM3/T
       TE(NROW,NCOL)=DUM3*DUM1+TE(NROW,NCOL)
200     CONTINUE
C
C*****************************************************
C    THE CONVECTIVE BOUNDARIES ARE HANDLED
C
      IF(NCONV.EQ.0) GO TO 400
       IF(MEQ.EQ.1) GO TO 400
       DO 301 I=1,NCONV
     II=I
       IF(LLL.EQ.NCON(I,1)) CALL CONVEC
 300   CONTINUE
```

```
301    CONTINUE                                                         GAU12700
400    CONTINUE                                                         GAU12800
       VYY=(AVY(1)+AVY(2)+AVY(3)+AVY(4))/4                              GAU12900
      IF(KBOUND.EQ.99) RETURN                                           GAU13000
      IF(TT.EQ.0) RETURN                                                GAU13100
C************************************************************           GAU13200
C        CRANK-NICOLSON METHOD FOR TREATING TRANSIENT CONDITIONS        GAU13300
C                                                                       GAU13400
C                                                                       GAU13500
      DO 450 NROW=1,M4                                                  GAU13600
      GE(NROW)=GE(NROW)*ALPHA                                           GAU13700
      DO 450 NCOL=1,M4                                                  GAU13800
      DUM4=SE(NROW,NCOL)                                                GAU13900
      SE(NROW,NCOL)=ALPHA/2*SE(NROW,NCOL)+TE(NROW,NCOL)                 GAU14000
      TE(NROW,NCOL)=TE(NROW,NCOL)-ALPHA/2*DUM4                          GAU14100
 450  CONTINUE                                                          GAU14200
800    CONTINUE                                                         GAU14300
C************************************************************           GAU14400
       RETURN                                                           GAU14500
C      ROUTINE THAT ADDS IN CONVECTIVE BOUNDARY TERMS                   GAU14600
C                                                                       GAU14700
       SUBROUTINE CONVEC                                                GAU14800
       LA=NCON(I,2)                                                     GAU14900
       MC1=AY(LA)                                                       GAU15000
       MC2=AZ(LA)                                                       GAU15100
       A=XL(MC1)-XL(MC2)                                                GAU15200
       MC3=MC1+MC2                                                      GAU15300
       IF(MC3.EQ.5) A=YL(MC1)-YL(MC2)                                   GAU15400
       A=ABS(A)                                                         GAU15500
       DO 505 J=1,4                                                     GAU15600
505    HE(J)=0.0                                                        GAU15700
       HE(MC1)=.5                                                       GAU15800
       HE(MC2)=.5                                                       GAU15900
       ALOC(II)=A*THICK*T                                               GAU16000
       CO=CONV(II)                                                      GAU16100
       IF(CO.LT.0.00001) GO TO 600                                      GAU16200
C                                                                       GAU16300
C                                                                       GAU16400
       CBAL(II,1)=ALPHA*CO*THICK*T*A                                    GAU16500
      DO 550 K=1,4                                                      GAU16600
      DO 550  J=1,4                                                     GAU16700
 550   SE(K,J)=HE(J)*HE(K)*CO*THICK*T*A+SE(K,J)                         GAU16800
C                                                                       GAU16900
       RETURN                                                           GAU17000
C**************************************************                     GAU17100
C                                                                       GAU17200
600    CONTINUE                                                         GAU17300
       GO TO (603,601,601,603),LA                                       GAU17400
601    DO 602 J=1,4                                                     GAU17500
       IF(MC3.EQ.5) VEL(J)=AVX(J)                                       GAU17600
602    IF(MC3.NE.5) VEL(J)=AVY(J)                                       GAU17700
       GO TO 605                                                        GAU17800
603    DO 604 J=1,4                                                     GAU17900
       IF(MC3.EQ.5) VEL(J)=-AVX(J)                                      GAU18000
604    IF(MC3.NE.5) VEL(J)=-AVY(J)                                      GAU18100
605    CONTINUE                                                         GAU18200
C                                                                       GAU18300
       CO=-CO                                                           GAU18400
       NCO=CO                                                           GAU18500
       VT=VEL(MC1)/2+VEL(MC2)/2                                         GAU18600
       IF(NCO.LT.995.OR.NCO.GT.1000) GO TO 700                          GAU18700
       IF(NCO.EQ.999) CO=VT*PCH*ERROR                                   GAU18800
       IF(NCO.EQ.996) CO=(-VT+VYY)*PCH*ERROR                            GAU18900
      IF(NCO.EQ.997) CO=ERROR                                           GAU19000
       CBAL(II,1)=ALPHA*CO*THICK*T*A                                    GAU19100
      DO 650 K=1,4                                                      GAU19200
      DO 650  J=1,4                                                     GAU19300
 650   SE(K,J)=HE(J)*HE(K)*CO*THICK*T*A+SE(K,J)                         GAU19400
        RETURN                                                          GAU19500
C                                                                       GAU19600
C*******************************************************                GAU19700
C                                                                       GAU19800
700    CONTINUE                                                         GAU19900
C CALCULATE HEAT TRANSFER OUT WITH FLOWING WATER                        GAU20000
C                                                                       GAU20100
       CBAL(II,1)=ALPHA*VT*PCH*THICK*T*A                                GAU20200
C                                                                       GAU20300
       DUM1=1-AA(1)                                                     GAU20400
       DUM2=1-AA(2)                                                     GAU20500
       HE(MC1)=DUM1/4                                                   GAU20600
       HE(MC2)=DUM2/4                                                   GAU20700
       VELL=VEL(MC1)                                                    GAU20800
       DO 720 K=1,4                                                     GAU20900
       DO 720 J=1,4                                                     GAU21000
720    SE(K,J)=HE(J)*HE(K)*VELL*PCH*THICK*T*A*2+SE(K,J)                 GAU21100
       HE(MC1)=DUM2/4                                                   GAU21200
       HE(MC2)=DUM1/4                                                   GAU21300
       VELL=VEL(MC2)                                                    GAU21400
       DO 730 K=1,4                                                     GAU21500
       DO 730 J=1,4                                                     GAU21600
730    SE(K,J)=HE(J)*HE(K)*VELL*PCH*THICK*T*A*2+SE(K,J)                 GAU21700
       RETURN                                                           GAU21800
       END                                                              GAU21900
```

```
C*************************************************                        SH1  100
       SUBROUTINE SHAPE1(II,JJ)                                           SH1  200
C**********************************************                           SH1  300
      REAL JAC                                                            SH1  400
C                                                                         SH1  500
       DOUBLE PRECISION AA,NOT,NXI,NET,SE,TE,BB ,DUM1,DUM2                SH1  600
       COMMON/AA/NXI(12),NET(12),NOT(12)                                  SH1  700
       COMMON/AAA/XL(4),YL(4),DETJAC,JAC(4,4)                             SH1  800
       COMMON/AB/AA(6),W(4)/ABB/SE(12,12),TE(12,12)                       SH1  900
       COMMON/AC/RE(12),GE(12)/ACC/KK(12),K1,K2,K3,K4                     SH1 1000
      COMMON/HH/LFLOW,LON/HM/PXX,PYY,PXY,KAD                              SH1 1100
      COMMON/AT/XII(4),ETI(4)/CON/MC1,MC2,NCONV/AS/BB(2,12)               SH1 1200
C   COMPUTE THE SHAPE FUNCTIONS AND THEIR DERIVATIVES                     SH1 1300
      DO 100 I=1,4                                                        SH1 1400
      DUM1=(1.+XII(I)*AA(II))*.25                                         SH1 1500
      DUM2=(1.+ETI(I)*AA(JJ))*.25                                         SH1 1600
         NXI(I)=XII(I)*DUM2                                               SH1 1700
      NET(I)=ETI(I)*DUM1                                                  SH1 1800
       NOT(I)=4.*DUM1*DUM2                                                SH1 1900
  100  CONTINUE                                                           SH1 2000
C                                                                         SH1 2100
C**********************************************                           SH1 2200
C   COMPUTE JACOBIAN, AND ITS DETERMINANT                                 SH1 2300
      DO 150 I=1,2                                                        SH1 2400
      DO 150 J=1,2                                                        SH1 2500
  150 JAC(I,J)=0.0                                                        SH1 2600
      DO 160 I=1,4                                                        SH1 2700
      JAC(1,1)=JAC(1,1)+NXI(I)*XL(I)                                      SH1 2800
       JAC(1,2)=JAC(1,2)+NXI(I)*YL(I)                                     SH1 2900
      JAC(2,1)=JAC(2,1)+NET(I)*XL(I)                                      SH1 3000
      JAC(2,2)=JAC(2,2)+NET(I)*YL(I)                                      SH1 3100
  160  CONTINUE                                                           SH1 3200
      DETJAC=JAC(1,1)*JAC(2,2)-JAC(2,1)*JAC(1,2)                          SH1 3300
      DUM1=JAC(1,1)/DETJAC                                                SH1 3400
      JAC(1,1)=JAC(2,2)/DETJAC                                            SH1 3500
      JAC(1,2)=-JAC(1,2)/DETJAC                                           SH1 3600
      JAC(2,1)=-JAC(2,1)/DETJAC                                           SH1 3700
      JAC(2,2)=DUM1                                                       SH1 3800
        DO 170 L=1,4                                                      SH1 3900
      BB(1,L)=JAC(1,1)*NXI(L)+JAC(1,2)*NET(L)                             SH1 4000
      BB(2,L)=JAC(2,1) *NXI(L)+JAC(2,2)*NET(L)                            SH1 4100
170     CONTINUE                                                          SH1 4200
       RETURN                                                             SH1 4300
       END                                                                SH1 4400
      SUBROUTINE SHAPE(L,M,XI,YI,X,Y,NN,NE,IN)
C     ******************************************
C
C  XI/YI THE GAUSS POINTS
C  X/Y THE X AND Y LOCATIONS OF THE ELEMENT NODES
C  NN  NUMBER OF NODES
C  NE  NUMBER OF ELEMENTS
C  IN ARRAY REFERED TO IN MAIN PROGRAM AS NOD
C  M REFERED TO IN MAIN PROGRAM AS M4, NUMBER OF NODES IN THE ELEMENT[
C  DET IS THE DETERMINENT, ONE QUARTER OF THE AREA OF THE ELEMENT
C
C
C     CALLED FROM MNPGM
C     PURPOSE:  TO COMPUTE BASIS FUNCTIONS
C     ORIGINALLY PROGRAMED BY EMIL O. FRIND
C     ------------------------------------------------------------------
               DOUBLE PRECISION  DFX,DFY,FF,BB
      DIMENSION X(NN), Y(NN), IN(12,NE)
      DIMENSION ALF(4), DAX(4), DAY(4), BTX(4), BTY(4), DBX(4),
     1 DBY(4)
C
       COMMON/AA/DFX(12),DFY(12),FF(12)
     */AAA/ZZZ(4),ZZZZZ(4),DET,DJAC(4,4)
       COMMON/AS/BB(2,12)
C     ------------------------------------------------------------------
C
C     XI/YI - LOCAL COORDINATES OF THE INTEGRATION POINTS
      XI1=1.-XI
      XI2=1.+XI
      YI1=1.-YI
      YI2=1.+YI
C
C     CORNER NODE SHAPE FUNCTIONS, BASIC PART
C     ALF - ALPHA PART OF SHAPE FUNCTION
      ALF(1)=.25*XI1*YI1
      ALF(2)=.25*XI2*YI1
      ALF(3)=.25*XI2*YI2
      ALF(4)=.25*XI1*YI2
C     DAX/DAY - X- AND Y-DERIVATIVE OF ALPHA PART
      DAX(1)=-.25*YI1
      DAX(2)=.25*YI1
      DAX(3)=.25*YI2
      DAX(4)=-.25*YI2
      DAY(1)=-.25*XI1
      DAY(2)=-.25*XI2
      DAY(3)=.25*XI2
      DAY(4)=.25*XI1
C
C     CORNER NODE SHAPE FUNCTIONS, SIDE-DEPENDENT PART
      XQ1=XI-.5
      XQ2=-XI-.5
      YQ1=YI-.5
      YQ2=-YI-.5
      XC1=1.125*XI*XI-.625
      XC2=2.25*XI
      YC1=1.125*YI*YI-.625
      YC2=2.25*YI
      J1=1
      J2=2
      J3=5
C     FOR BETA X PART (BTX) OF SHAPE FUNCTION
      DO 50 J=1,2
      IF (IN(J3,L).EQ.0) GO TO 10
      IF (IN(J3+1,L).EQ.0) GO TO 20                         SHA 6400
      GO TO 30                                              SHA 6500
   10 CONTINUE                                              SHA 6600
C     LINEAR . . .                                          SHA 6700
      BTX(J1)=.5                                            SHA 6800
      BTX(J2)=.5                                            SHA 6900
C     DBX - BETA X DERIVATIVE                               SHA 7000
      DBX(J1)=0.                                            SHA 7100
      DBX(J2)=0.                                            SHA 7200
      GO TO 40                                              SHA 7300
   20 CONTINUE                                              SHA 7400
C     QUADRATIC . . .                                       SHA 7500
      BTX(J1)=XQ2                                           SHA 7600
      BTX(J2)=XQ1                                           SHA 7700
      DBX(J1)=-1.                                           SHA 7800
      DBX(J2)=1.                                            SHA 7900
      GO TO 40                                              SHA 8000
   30 CONTINUE                                              SHA 8100
C     CUBIC . . .                                           SHA 8200
      BTX(J1)=XC1                                           SHA 8300
      BTX(J2)=XC1                                           SHA 8400
      DBX(J1)=XC2                                           SHA 8500
      DBX(J2)=XC2                                           SHA 8600
   40 CONTINUE                                              SHA 8700
      J1=4                                                  SHA 8800
      J2=3                                                  SHA 8900
      J3=9                                                  SHA 9000
   50 CONTINUE                                              SHA 9100
      J1=2                                                  SHA 9200
      J2=3                                                  SHA 9300
      J3=7                                                  SHA 9400
C     FOR BETA Y PART (BTY) OF SHAPE FUNCTION               SHA 9500
      DO 100 J=1,2                                          SHA 9600
      IF (IN(J3,L).EQ.0) GO TO 60                           SHA 9700
      IF (IN(J3+1,L).EQ.0) GO TO 70                         SHA 9800
      GO TO 80                                              SHA 9900
   60 CONTINUE                                              SHA10000
C     LINEAR . . .                                          SHA10100
      BTY(J1)=.5                                            SHA10200
      BTY(J2)=.5                                            SHA10300
C     DBY - BETA Y DERIVATIVE                               SHA10400
      DBY(J1)=0.                                            SHA10500
      DBY(J2)=0.                                            SHA10600
      GO TO 90                                              SHA10700
   70 CONTINUE                                              SHA10800
C     QUADRATIC . . .                                       SHA10900
      BTY(J1)=YQ2                                           SHA11000
      BTY(J2)=YQ1                                           SHA11100
      DBY(J1)=-1.                                           SHA11200
      DBY(J2)=1.                                            SHA11300
      GO TO 90                                              SHA11400
   80 CONTINUE                                              SHA11500
C     CUBIC . . .                                           SHA11600
      BTY(J1)=YC1                                           SHA11700
      BTY(J2)=YC1                                           SHA11800
      DBY(J1)=YC2                                           SHA11900
      DBY(J2)=YC2                                           SHA12000
   90 CONTINUE                                              SHA12100
      J1=1                                                  SHA12200
      J2=4                                                  SHA12300
      J3=11                                                 SHA12400
  100 CONTINUE                                              SHA12500
C                                                           SHA12600
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C     SHAPE FUNCTION DERIVATIVE MATRIX - CORNER NODES                   SHA12700
      DO 110 J=1,4                                                      SHA12800
      DFX(J)=DAX(J)*(BTX(J)+BTY(J))+DBX(J)*ALF(J)                       SHA12900
      DFY(J)=DAY(J)*(BTX(J)+BTY(J))+DBY(J)*ALF(J)                       SHA13000
      FF(J)=ALF(J)*(BTX(J)+BTY(J))                                      SHA13100
  110 CONTINUE                                                          SHA13200
C                                                                       SHA13300
C     SHAPE FUNCTION DERIVATIVE MATRIX - EDGE NODES                     SHA13400
C     SKIP IF ELEMENT IS LINEAR (M=4)                                   SHA13500
      IF (M.EQ.4) GO TO 240                                             SHA13600
      J=4                                                               SHA13700
      XEQ=1.-XI*XI                                                      SHA13800
      YEQ=1.-YI*YI                                                      SHA13900
      XE1=1.-3.*XI                                                      SHA14000
      XE2=1.+3.*XI                                                      SHA14100
      YE1=1.-3.*YI                                                      SHA14200
      YE2=1.+3.*YI                                                      SHA14300
      IF (IN(5,L).EQ.0) GO TO 140                                       SHA14400
      IF (IN(6,L).EQ.0) GO TO 120                                       SHA14500
      GO TO 130                                                         SHA14600
  120 J=J+1                                                             SHA14700
      DFX(J)=-XI*YI1                                                    SHA14800
      DFY(J)=-.5*XEQ                                                    SHA14900
      FF(J)=.5*XEQ*YI1                                                  SHA15000
      GO TO 140                                                         SHA15100
  130 J=J+1                                                             SHA15200
      DFX(J)=-.28125*YI1*(3.*XEQ+2.*XI*XE1)                             SHA15300
      DFY(J)=-.28125*XEQ*XE1                                            SHA15400
      FF(J)=.28125*XEQ*XE1*YI1                                          SHA15500
      J=J+1                                                             SHA15600
      DFX(J)=.28125*YI1*(3.*XEQ-2.*XI*XE2)                              SHA15700
      DFY(J)=-.28125*XEQ*XE2                                            SHA15800
      FF(J)=.28125*XEQ*XE2*YI1                                          SHA15900
  140 IF (IN(7,L).EQ.0) GO TO 170                                       SHA16000
      IF (IN(8,L).EQ.0) GO TO 150                                       SHA16100
      GO TO 160                                                         SHA16200
  150 J=J+1                                                             SHA16300
      DFX(J)=.5*YEQ                                                     SHA16400
      DFY(J)=-YI*XI2                                                    SHA16500
      FF(J)=.5*XI2*YEQ                                                  SHA16600
      GO TO 170                                                         SHA16700
  160 J=J+1                                                             SHA16800
      DFX(J)=.28125*YEQ*YE1                                             SHA16900
      DFY(J)=-.28125*XI2*(3.*YEQ+2.*YI*YE1)                             SHA17000
      FF(J)=.28125*XI2*YEQ*YE1                                          SHA17100
      J=J+1                                                             SHA17200
      DFX(J)=.28125*YEQ*YE2                                             SHA17300
      DFY(J)=.28125*XI2*(3.*YEQ-2.*YI*YE2)                              SHA17400
      FF(J)=.28125*XI2*YEQ*YE2                                          SHA17500
  170 IF (IN(9,L).EQ.0) GO TO 200                                       SHA17600
      IF (IN(10,L).EQ.0) GO TO 180                                      SHA17700
      GO TO 190                                                         SHA17800
  180 J=J+1                                                             SHA17900
      DFX(J)=-XI*YI2                                                    SHA18000
      DFY(J)=.5*XEQ                                                     SHA18100
      FF(J)=.5*XEQ*YI2                                                  SHA18200
      GO TO 200                                                         SHA18300
  190 J=J+1                                                             SHA18400
      DFX(J)=.28125*YI2*(3.*XEQ-2.*XI*XE2)                              SHA18500
      DFY(J)=.28125*XEQ*XE2                                             SHA18600
      FF(J)=.28125*XEQ*XE2*YI2                                          SHA18700
      J=J+1                                                             SHA18800
      DFX(J)=-.28125*YI2*(3.*XEQ+2.*XI*XE1)                             SHA18900
      FF(J)=.28125*XEQ*XE1*YI2                                          SHA19000
      DFY(J)=.28125*XEQ*XE1                                             SHA19100
  200 IF (IN(11,L).EQ.0) GO TO 230                                      SHA19200
      IF (IN(12,L).EQ.0) GO TO 210                                      SHA19300
      GO TO 220                                                         SHA19400
  210 J=J+1                                                             SHA19500
      DFX(J)=-.5*YEQ                                                    SHA19600
      DFY(J)=-YI*XI1                                                    SHA19700
      FF(J)=.5*XI1*YEQ                                                  SHA19800
      GO TO 230                                                         SHA19900
  220 J=J+1                                                             SHA20000
      DFX(J)=-.28125*YEQ*YE2                                            SHA20100
      DFY(J)=.28125*XI1*(3.*YEQ-2.*YI*YE2)                              SHA20200
      FF(J)=.28125*XI1*YEQ*YE2                                          SHA20300
      J=J+1                                                             SHA20400
      DFX(J)=-.28125*YEQ*YE1                                            SHA20500
      DFY(J)=-.28125*XI1*(3.*YEQ+2.*YI*YE1)                             SHA20600
      FF(J)=.28125*YEQ*YE1*XI1                                          SHA20700
  230 CONTINUE                                                          SHA20800
  240 CONTINUE                                                          SHA20900
C                                                                       SHA21000
C     JACOBIAN                                                          SHA21100
      SUM1=0.                                                           SHA21200
      SUM2=0.                                                           SHA21300
      SUM3=0.                                                           SHA21400
      SUM4=0.                                                           SHA21500
      K=0                                                               SHA21600
      DO 260 I=1,M                                                      SHA21700
  250 K=K+1                                                             SHA21800
      IF (IN(K,L).EQ.0) GO TO 250                                       SHA21900
      KI=IN(K,L)                                                        SHA22000
      SUM1=SUM1+DFX(I)*X(KI)                                            SHA22100
      SUM2=SUM2+DFX(I)*Y(KI)                                            SHA22200
      SUM3=SUM3+DFY(I)*X(KI)                                            SHA22300
      SUM4=SUM4+DFY(I)*Y(KI)                                            SHA22400
  260 CONTINUE                                                          SHA22500
      DET=SUM1*SUM4-SUM2*SUM3                                           SHA22600
      DET1=1./DET                                                       SHA22700
      C11=DET1*SUM4                                                     SHA22800
      C12=-DET1*SUM2                                                    SHA22900
      C21=-DET1*SUM3                                                    SHA23000
      C22=DET1*SUM1                                                     SHA23100
C                                                                       SHA23200
                   DJAC(1,1)=C11                                        SHA23300
                   DJAC(1,2)=C12                                        SHA23400
                   DJAC(2,1)=C21                                        SHA23500
                   DJAC(2,2)=C22                                        SHA23600
C     SHAPE FUNCTION DERIVATIVES - GLOBAL                               SHA23700
      DO 270 J=1,M                                                      SHA23800
      BB(1,J)=C11*DFX(J)+C12*DFY(J)                                     SHA23900
      BB(2,J)=C21*DFX(J)+C22*DFY(J)                                     SHA24000
  270 CONTINUE                                                          SHA24100
      RETURN                                                            SHA24200
      END                                                               SHA24300
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C***********************************************************************
C
C    THIS ROUTINE COMPUTES THE COLUMN MATRIX R BY ADDING IN THE VARIOUS
C       SOURCES AND SINKS AND THEN SOLVES FOR THE UNKNOWN COLUMN MATRIX
C
C
      SUBROUTINE LOAD(LM,LN,MBAND,NOD,NODE,R,RI,G
     *,GG,HEAT,HEAD,ALPHA,KBOUND,KFT,PCW,LT,CFACT,KS,
     *Z,NCON,TINF,CONV,ALOC,LSOURC,NSOURC,ASOURC,CBAL,AY,AZ
     * ,AEL,NEL,MEQ,LEL,CBALA,LINE,NLINE,ALINE)
       REAL JAC
       INTEGER Z,AY(4),AZ(4)
      DOUBLE PRECISION AA,NET,NXI,NOT
       COMMON/AAA/XL(4),YL(4),DETJAC,JAC(4,4)/AB/AA(6),W(4)
       COMMON/A1/M,MM,NUMNP,NSIZE/AA/NXI(12),NET(12),NOT(12)
       COMMON/ACC/KK(12),K1,K2,K3,K4/AM/KSYM,PCT/HH/LFLOW,LON
      COMMON/BAL/BLINE/CON/MC1,MC2,NCONV/CONT/LA,LB,LC,LD,LE,LF,LG,LH
       COMMON/ATHICK/ASIZE,NTHICK,THICK,ERROR,XADD,YADD
      DIMENSION NCON(Z,3),TINF(Z),CONV(Z),ALOC(Z),NSOURC(Z)
      DIMENSION AEL(Z,2), CBAL(Z,2), ASOURC(Z),NODE(LM),NOD(12,LM)
        DIMENSION R(LN),G(LN),GG(LN),RI(LN),HEAD(LN),HEAT(LM)
     *,NLINE(Z,2),ALINE(Z),NEL(Z,2)
        RETURN
C
C***********************************************************************
C
          ENTRY HEATE(KS)
C
C
C
C
C
C
        DO 1 J=1,LN
        RI(J)=R(J)
        G(J)=0.0
1       CONTINUE
       BLINE=0.0
       IF(LD.EQ.1.AND.MEQ.EQ.1) CALL CHAN
       IF(LT.EQ.0) GO TO 300
C***********************************************************************
C
C  TRANSIENT LOOP  GENERATION AND BOUNDARY CONDITIONS ARE CHANGED AT EAC
       IF(KBOUND.EQ.1) CALL BOUND(KS)
C THREE ENTRY POINTS ARE PROVIDED FOR CHANGING RECHARGE RATES
C AND BOUNDARY CONDITIONS AT EACH TIME STEP--ALL ENTRIES IN BOUNDA
C
       IF(KBOUND.EQ.2) CALL BVAL(KS,ALPHA)
C
       IF(LD.EQ.1.AND.MEQ.EQ.2) CALL CHANG
C
C***********************************************************************
C
C***********************************************************************
C     CONVECTIVE BOUNDARY ROUTINE
       IF(MEQ.EQ.1) GO TO 50
C
      IF(NCONV.EQ.0) GO TO 40
C
       DO 30   I=1,NCONV
      LLL=NCON(I,1)
       LA=NCON(I,2)
       MC1=AY(LA)
       MC2=AZ(LA)
       MA=NOD(MC1,LLL)
       NA=NOD(MC2,LLL)
       K=NA
       ONV=ABS(CONV(I))
       CBAL(I,2)=ONV*ALOC(I)*TINF(I)*ALPHA
C   H*LENGTH PF BOUNDARY*TEMP AT INFINITY*TIME STEP
      G(K)=ONV*ALOC(I)*.5*TINF(I)*ALPHA+G(K)
       K=MA
      G(K)=ONV*ALOC(I)*.5*TINF(I)*ALPHA+G(K)
 30   CONTINUE
C
 40   CONTINUE
C
C ROUTINE THAT CALCUALTES CONVECTIVE INPUTS IS CALLED
       IF(LEL.GT.0) CALL CB(ALPHA)
C
C
50     CONTINUE
C***********************************************************************
C--MATRIX MULTIPLICATION LOOP
C
        IF(KSYM.EQ.0) CALL MULTI
       IF(KSYM.NE.0) CALL AMULTI
C
C***********************************************************************
C
C
C  GENERATION TERMS ARE ADDED INTO THE RIGHT SIDE OF THE EQUATION
C
C
C GENERATION TERMS FROM POINT SOURCES
C
300      IF(LSOURC.EQ.0) GO TO 360
         DO 350 LLL=1,LSOURC
        K=NSOURC(LLL)
      IF(ALPHA.LE.0) AS=ASOURC(LLL)
      IF(ALPHA.GT.0) AS=ASOURC(LLL)*ALPHA
      IF(LT.EQ.0) GG(K)=GG(K)+AS
      IF(LT.EQ.1) G(K)=G(K)+AS
350   CONTINUE
360   CONTINUE
C
C  GENERATION TERMS FROM LINE SOURCES ARE COMPUTED AND ADDED IN
C
      IF(LINE.EQ.0) GO TO 378
      DO 375 L=1,LINE
      LLL=NLINE(L,1)
      CALL CORD(LLL)
      LA=NLINE(L,2)
      LY=AY(LA)
      LZ=AZ(LA)
      MC3=LY+LZ
      IF(MC3.EQ.5) A=ABS(YL(LY)-YL(LZ))
      IF(MC3.NE.5) A=ABS(XL(LY)-XL(LZ))
      AS=ALINE(L)*A
      IF(ALPHA.GT.0) AS=AS*ALPHA
      BLINE=BLINE+AS
      LY=NOD(LY,LLL)
      LZ=NOD(LZ,LLL)
      IF(LT.EQ.0) GO TO 372
      G(LY)=G(LY)+AS/2
      G(LZ)=G(LZ)+AS/2
      GO TO 375
372   GG(LY)=GG(LY)+AS/2
      GG(LZ)=GG(LZ)+AS/2
375   CONTINUE
C
378   CONTINUE
C
      DO 380 N=1,NSIZE
380    R(N)=R(N)+G(N)
C
C***********************************************************************
C
C     GENERATION TERMS FROM SOURCES ARE ADDED IN
C    AND SPECIFIED HEADS ARE TREATED
C
        DO 410 N=1,NSIZE
       IF(LT.EQ.0) R(N)=CFACT*HEAD(N)+GG(N)
 410   IF(LT.EQ.1) R(N)=R(N)+CFACT*HEAD(N)+GG(N)
C
C***********************************************************************
C
C  FORWARD REDUCTION OF THE LOAD MATRIX IS PREFORMED AND BACK
C    SUBSTITUTION IS PREFORMED TO SOLVE FOR THE UNKNOWN MATRIX
C
          IF(KSYM.EQ.0)  CALL BACK
          IF(KSYM.NE.0) CALL ABACK
550       CONTINUE
C
C*******************************************************************
       RETURN
C
C *****************************************************************
C
C
         SUBROUTINE CB(ALPHA)
C
C******************************************************************
         DIMENSION JA(4),JB(4),JC(4),JD(4)
         DATA JA/1,1,1,2/JB/2,1,2,2/JC/2,2,1,2/JD/2,1,1,1/
         IF(LEL.EQ.0) RETURN
         AA(1)=-(1/3)**.5
         AA(2)=-AA(1)
         FACT1=0.7886751
         FACT2=0.2113249
         CBALA=0.0
         LFLOW=0
         DO 10 J=1,LEL
         LLL=NEL(J,1)
         M4=4
         CALL DERIVE(LLL,M4)
         T=1.0
         CALL PEE(T,LLL)
         LA=NEL(J,2)
         LY=AY(LA)
          LZ=AZ(LA)
         IF(LA.GT.2) A=YL(LY)-YL(LZ)
         IF(LA.LT.3) A=XL(LY)-XL(LZ)
         A=ABS(A)*T*THICK
         LY=NOD(LY,LLL)
         LZ=NOD(LZ,LLL)
C
C   COMPUTE VELOCITIES AT THE GAUSS POINTS
```

```
C
C     VX IS ASSOCIATED WITH ARRAY AY       VXX IS ASSOCIATED WITH ARRAY AZ
      JJ=JA(LA)
      II=JB(LA)
      CALL SHAPE1(JJ,II)
      CALL VELO(LLL,VX,VY)
      JJ=JC(LA)
      II=JD(LA)
      CALL SHAPE1(JJ,II)
      CALL VELO(LLL,VXX,VYY)
C
      IF(LA.GT.2) GO TO 6
      V1=(VY*A/2)
      V2=(VYY*A/2)
      GO TO 7
6     V1=(VX*A/2)
      V2=(VXX*A/2)
7     CONTINUE
      GO TO (9,8,8,9),LA
8     V1=-V1
      V2=-V2
9     CONTINUE
      IF(V1.GE.0) Q=V1*PCW*AEL(J,2)
      IF(V1.LT.0)  Q=V1*PCW*R(LY)
      IF(V2.GE.0) QQ=V2*PCW*AEL(J,2)
      IF(V2.LT.0) QQ=V2*PCW*R(LZ)
      G(LY)=G(LY)+Q*ALPHA*FACT1+QQ*ALPHA*FACT2
      G(LZ)=G(LZ)+QQ*ALPHA*FACT1+Q*ALPHA*FACT2
      CBALA=CBALA+Q+QQ
10    CONTINUE
20    FORMAT(1X,'BOUND TEMPS  ',7(I4,G8.3,3X))
      RETURN
      END
```

```
C***********************************************************************
C
C     FORWARD REDUCTION OF THE S MATRIX BY GAUSS ELIMINATION
C
      SUBROUTINE SOLVE(LN,MBAND,S,T,R,RI)
C
      DOUBLE PRECISION S,T,C
      COMMON/A1/ M,MM,NUMNP,NSIZE
      DIMENSION S(LN,MBAND),T(LN,MBAND),R(LN),RI(LN)
C GAUSS ELIMINATION--FORWARD REDUCTION OF STIFNESS MATRIX
      DO 550 N=1,NSIZE
      DO 530 L=2,MBAND
      IF(S(N,L).EQ.0) GO TO 530
      I=N+L-1
      C=S(N,L)/S(N,1)
       J=0
      DO 510 K=L,MBAND
      J=J+1
 510  S(I,J)=S(I,J)-C*S(N,K)
      S(N,L)=C
530    CONTINUE
550    CONTINUE
      RETURN
C
C***********************************************************************
C     BACK SOLUTION OF THE R VECTOR BY GAUSS ELIMINATION
C
      ENTRY BACK
C
C
C  FORWARD REDUCTION OF THE LOAD MATRIX IS PREFORMED AND BACK
C    SUBSTITUTION IS PREFORMED TO SOLVE FOR THE UNKNOWN MATRIX
      DO 630 N=1,NSIZE
      DO 620 L=2,MBAND
      IF(S(N,L).EQ.0) GO TO 620
      I=N+L-1
      R(I)=R(I)-S(N,L)*R(N)
620   CONTINUE
630   R(N)=R(N)/S(N,1)
      DO 660 M=2,NSIZE
      N=NSIZE+1-M
      DO 650 L=2,MBAND
      IF(S(N,L).EQ.0) GO TO 650
      K=N+L-1
      R(N)=R(N)-S(N,L)*R(K)
650      CONTINUE
660    CONTINUE
C
      RETURN
C
C***********************************************************************
C  MULTIPLICATION OF THE T MATRIX BY THE NODAL VALUES AT THE LAST TIME S
C
C
      ENTRY MULTI
C
C    MATRIX MULTIPLICATION LOOP--CRANK-NICOLSON METHOD
C
       DO 150 N=1,NSIZE
       R(N)=0.0
      DO 110 M=1,MBAND
      NO=M+N-1
      IF(NO.GT.NSIZE) GO TO 120
```

```
  110  R(N)=RI(NO)*T(N,M)+R(N)                                         SOL 6400
  120  CONTINUE                                                        SOL 6500
      MBA=MBAND-1                                                      SOL 6600
       DO 130 M=1,MBA                                                  SOL 6700
      NO=N-M                                                           SOL 6800
      IF(NO.LE.0) GO TO 140                                            SOL 6900
  130  R(N)=RI(NO)*T(NO,M+1)+R(N)                                      SOL 7000
  140  CONTINUE                                                        SOL 7100
150   CONTINUE                                                         SOL 7200
C                                                                      SOL 7300
      RETURN                                                           SOL 7400
      END                                                              SOL 7500
```

```
C***********************************************************************
C
      SUBROUTINE ASOLVE(B,R,RI,NEQ,IHALFB,NDIM,MDIM,T)
C
C   ASSYMMETRIC BAND MATRIX EQUATION SOLVER
C    ORIGINALLY PROGRAMED BY J.O. DUGUID
C
      DOUBLE PRECISION B,T,PIVOT,C
      DIMENSION B(NDIM,MDIM),R(NDIM),T(NDIM,MDIM)
      DIMENSION RI(NDIM)
      NRS=NEQ-1
      IHBP=IHALFB+1
C
C  TR1ANGULARIZE MATRIX B USING DOOLITTLE METHOD
C
      DO 20 K=1,NRS
       PIVOT=B(K,IHBP)
      KK=K+1
      KC=IHBP
      DO 10 I=KK,NEQ
      KC=KC-1
      IF(KC.LE.0) GO TO 20
      C=-B(I,KC)/PIVOT
      B(I,KC)=C
      KI=KC+1
      LIM=KC+IHALFB
      DO 10 J=KI,LIM
      JC=IHBP+J-KC
10    B(I,J)=B(I,J)+C*B(K,JC)
20    CONTINUE
       RETURN
C
C***********************************************************************
C
      ENTRY ABACK
C
      NN=NEQ+1
      IBAND=2*IHALFB+1
      DO 70 I=2,NEQ
      JC=IHBP-I+1
      JI=1
      IF(JC.LE.0) GO TO 40
      GO TO 50
40    JC=1
      JI=I-IHBP+1
50    SUM=0.0
      DO 60 J=JC,IHALFB
      SUM=SUM+B(I,J)*R(JI)
60    JI=JI+1
70    R(I)=R(I)+SUM
C
C   BACK SUBSTITUTION
C
      R(NEQ)=R(NEQ)/B(NEQ,IHBP)
      DO 90 IBACK=2,NEQ
      I=NN-IBACK
      JP=I
      KR=IHBP+1
      MR=MIN0(IBAND,IHALFB+IBACK)
      SUM=0.0
      DO 80 J=KR,MR
      JP=JP+1
```

```
80    SUM=SUM+B(I,J)*R(JP)                                              ASO 6300
90    R(I)=(R(I)-SUM)/B(I,IHBP)                                         ASO 6400
100   RETURN                                                            ASO 6500
C                                                                       ASO 6600
C***********************************************************************ASO 6700
C                                                                       ASO 6800
      ENTRY AMULTI                                                      ASO 6900
C                                                                       ASO 7000
      LN=NDIM                                                           ASO 7100
      MBAND=(MDIM+1)/2                                                  ASO 7200
      DO 150 N=1,LN                                                     ASO 7300
      R(N)=0.0                                                          ASO 7400
      DO 110 M=1,MDIM                                                   ASO 7500
      NO=M+N-MBAND                                                      ASO 7600
      IF(NO.GT.LN) GO TO 120                                            ASO 7700
      IF(NO.LT.1) GO TO 110                                             ASO 7800
      R(N)=RI(NO)*T(N,M)+R(N)                                           ASO 7900
110     CONTINUE                                                        ASO 8000
120   CONTINUE                                                          ASO 8100
150    CONTINUE                                                         ASO 8200
      RETURN                                                            ASO 8300
      END                                                               ASO 8400
```

```
C**************************************** ***************************
C    THIS SUBROUTINE COMPUTES A MASS BALANCE
C
      SUBROUTINE BALAN(LM,LN,R,PX,PY,PCX,HEAD,HEAT,Z,LSOURC,ASOURC,
     * LFLUX,NFLUX,A,B,C,D,E,MPRINT,NTYPE,RI,NODE,NOD,FLOWX,FLOWY,PCW
     *   ,MEQ,CBAL,CBALA,AY,AZ,NCON,KRAN)
      REAL  JAC
      INTEGER Z,AY,AZ
       DOUBLE PRECISION AA,NXI,NET.NOT
       COMMON/AAA/XL(4),YL(4),DETJAC,JAC(4.4)/AB/AA(6).W(4)
       COMMON/CON/M1,M2,NCONV/AA/NXI(12),NET(12),NOT(12)
       COMMON/ACC/KK(12),K1,K2,K3,K4/HH/LFLOW,LON/BAL/BLINE
      DIMENSION RI(LN),R(LN),PX(LM),PY(LM),NODE(LM)
       DIMENSION FLOWX(LM),FLOWY(LM),AY(4),AZ(4)
       DIMENSION NCON(Z,2),CBAL(Z,2) , NOD(12,LM)
      DIMENSION PCX(LM),HEAD(LN),HEAT(LM),ASOURC(Z).NFLUX(Z,2)
       DIMENSION LOA(4),LOB(4),LOC(4),LOD(4)
       DATA LOA/1,2,3,4/LOB/2,3.4,1/LOC/5,7.9,11/LOD/6 8.10,12/
      RETURN
C**************************************** ***************************
      ENTRY MASBAL(ALPHA)
C
C---COMPUTE BOUNDARY FLUXES
C
       ST=0.0
       PUMP=0.0
       F=0.0
       LFLOW=0
       RECH=0.0
       DISCH=0.0
       IF(LFLUX.EQ.0).GO TO 15
       AA(1)=0.0
       AA(2)=0.0
C
      DO 10  LLA=1,LFLUX
      LLL =NFLUX(LLA,1)
      K=NFLUX(LLA,2)
C
       CALL FLOWW(LLL)
C
       LT=NFLUX(LLA,2)
       FLUXX=0.0
       FLUXY=0.0
      IF(LT.GT.2) FLUXX=DHX*ARX*PX(LLL)
      IF(LT.LT.3) FLUXY=DHY*ARY*PY(LLL)
       IF(MEQ.EQ.1) GO TO 8
       LY=AY(LT)
       LZ=AZ(LT)
       LY=NOD(LY,LLL)
       LZ=NOD(LZ,LLL)
       RR=(R(LY)+R(LZ))/2
       IF(LT.GT.2) FLUXX=FLUXX+FLOWX(LLL)*PCW*RR
       IF(LT.LT.3) FLUXY=FLUXY+FLOWY(LLL)*PCW*RR
8      CONTINUE
       IF(LT.EQ.1.AND.FLUXY.LT.0) DISCH=DISCH-FLUXY
       IF(LT.EQ.1.AND.FLUXY.GT.0) RECH=RECH+FLUXY
       IF(LT.EQ.2.AND.FLUXY.LT.0) RECH=RECH-FLUXY
       IF(LT.EQ.2.AND.FLUXY GT.0) DISCH=DISCH+FLUXY
       IF(LT.EQ.3.AND.FLUXX.LT.0) RECH=RECH-FLUXX
       IF(LT.EQ.3.AND.FLUXX.GT.0) DISCH=DISCH+FLUXX
       IF(LT.EQ.4.AND.FLUXX.LT.0) DISCH=DISCH-FLUXX
       IF(LT.EQ.4.AND.FLUXX.GT.0) RECH=RECH+FLUXX
10    CONTINUE
```

```
15    CONTINUE
C
C****************************************  ***************************
C   COMPUTE RATE OF CHANGE IN STORAGE
C
        IF(KRAN.EQ.0) GO TO 21
       ST=0.0
       DO 20 LLL=1,LM
        M4=4
        CALL DERIVE(LLL,M4)
                IF(MEQ.EQ.1) CALL PEE(T,LLL)
                IF(MEQ.EQ.1) DETJAC=DETJAC/T
        RB=0
        RA=0
        K=0
        DO 19 I=1,4
        L1=LOA(I)
        L2=LOB(I)
        L3=LOC(I)
        L4=LOD(I)
        L5=NOD(L1,LLL)
        L6=NOD(L2,LLL)
        L7=NOD(L3.LLL)
        L8=NOD(L4,LLL)
        IF(L7.GT.0) GO TO 23
        RA=RA+.5*R(L5)+.5*R(L6)
        RB=RB+.5*RI(L5)+.5*RI(L6)
        GO TO 19
23      IF(L8.GT.0) GO TO 24
        RA=RA+1/3*R(L5)+2/3*R(L7)+1/3*R(L6)
        RB=RB+1/3*RI(L5)+1/3*RI(L6)+2/3*RI(L7)
        GO TO 19
24      RA=RA+1/8*R(L5)+1/8*R(L6)+3/8*R(L7)+3/8*R(L8)
        RB=RB+1/8*RI(L5)+1/8*RI(L6)+3/8*RI(L7)+3/8*RI(L8)
19      CONTINUE
        RA=RA/4
        RB=RB/4
      ST=ST+(RA-RB)*PCX(LLL)*4*DETJAC
20    CONTINUE
21    CONTINUE
C
C****************************************  ***************************
C      ADD IN PUMPING SOURCES
C
      PUMP=0.0
       PUMP=PUMP+BLINE
      IF(LSOURC.EQ.0) GO TO 31
      DO 30 LLL=1,LSOURC
      PUMP=PUMP+ASOURC(LLL)
30     CONTINUE
31     CONTINUE
C     DISTRIBUTED SOURCES
        DO 40 LLL=1,LM
       IF(HEAT(LLL).EQ.0) GO TO 40
        M4=NODE(LLL)
       CALL DERIVE(LLL,M4)
       PUMP=PUMP+HEAT(LLL)*4*DETJAC
40     CONTINUE
C
C ***********************************************************
C
```

```
C     ROUTINES TO COMPUTE FLUXES ACROSS CONVECTIVE BOUNDARIES
C
        IF(MEQ.EQ.1) GO TO 48
        IF(NCONV.EQ 0) GO TO 45
        CO=0.0
        COA=0.0
        DO 43 I=1,NCONV
        LLL=NCON(I,1)
        LA=NCON(I,2)
        MC1=AY(LA)
        MC2=AZ(LA)
        MA=NOD(MC1,LLL)
        NA=NOD(MC2,LLL)
        NC=CBAL(I,2)*100
        IF(NC.EQ.0) COA=COA+(R(MA)/4+R(NA)/4+RI(MA)/4+RI(NA)/4)*CBAL(I,1)
        IF(NC.EQ.0) GO TO 43
        CO=CO+CBAL(I,2)-(R(MA)/4+R(NA)/4+RI(MA)/4+RI(NA)/4)*CBAL(I,1)
43      CONTINUE
C
C
45      CONTINUE
48      CONTINUE
C*******************************************************
C
C
        AL=ALPHA
        IF(ALPHA.LT.0.0001) AL=1
        RECH=RECH*AL
        DISCH=DISCH*AL
        PUMP=PUMP*AL
         CBALA=CBALA*AL
C*****************************************************************
       A=A+RECH
       B=B+DISCH
       C=C+ST
       D=D+PUMP
        O=O+CO
        OO=OO+COA
       P=P+CBALA
       IF(MEQ.EQ.1) F=RECH-DISCH+PUMP-ST
       IF(MEQ.EQ.2) F=RECH-DISCH+PUMP-ST+CBALA+CO-COA
      E=E+F
        RETURN
        ENTRY BPRINT
       PRINT 50
         IF(MEQ.EQ.2) PRINT 51,O,CO.OO,COA,P,CBALA
      PRINT 52,A,RECH,B.DISCH,C,ST.D,PUMP,E,F
50     FORMAT(1X,T30,'CUMULATIVE MASS BLANCE'.
      *T60,'RATES FOR THIS TIME STEP')
51      FORMAT(1X,'CONDUCTIVE TRANSFER'.
      * ,T30,G12.6,T60,G12.6,
      * 1X/1X,'CONVECTIVE TRANSFER--OUT  ',T30,G12.6,T60,G12.6
      *,1X/1X,'CONVECTIVE TRANSFER--IN',T30,G12.6,T60,G12.6)
52      FORMAT(
      *1X,'B. FLUX RECHARGE',T30,G12.6,T60, G12.6
      */1X,'B. FLUX DISCHARGE',T30,G12.6,T60,G12.6,
      */1X,'CHANGE IN STORAGE',T30,G12.6,T60,G12.6.
      */1X,'QUANTITY PUMPED',T30.G12.6.T60,G12.6.
      */1X,'DIFFERECE',T30,G12.6,T60,G12.6)
       RETURN
C
```

```
C**************************************************************         BAL18600
C                                                                       BAL18700
       ENTRY WATER                                                      BAL18800
       AA(1)=0.0                                                        BAL18900
       AA(2)=0.0                                                        BAL19000
       LFLOW=0                                                          BAL19100
       DO 100 LLL=1,LM                                                  BAL19200
       CALL FLOWW(LLL)                                                  BAL19300
C                                                                       BAL19400
C     FLOW EQUALS THE SLOPE  *   AREA  * PERMEABILITY                   BAL19500
C                                                                       BAL19600
      FLOWX(LLL)=DHX*ARX*PX(LLL)                                        BAL19700
       FLOWY(LLL)=DHY*ARY*PY(LLL)                                       BAL19800
C                                                                       BAL19900
100    CONTINUE                                                         BAL20000
C ****************************************************************      BAL20100
       RETURN                                                           BAL20200
C                                                                       BAL20300
C                                                                       BAL20400
C ****************************************************************      BAL20500
C                                                                       BAL20600
       ENTRY VELOC(LN,LM,RW,WX WY)                                      BAL20700
       DIMENSION WX(LM),WY(LM),RW(LN)                                   BAL20800
       RETURN                                                           BAL20900
C***********************************************************************BAL21000
                ENTRY VELO(LLLQ,VX,VY)                                  BAL21100
       DHE=0.0                                                          BAL21200
       DHN=0.0                                                          BAL21300
       K=0                                                              BAL21400
       M4=NODE(LLLQ)                                                    BAL21500
       DO 500 I=1,M4                                                    BAL21600
400      K=K+1                                                          BAL21700
       IF(NOD(K,LLLQ).EQ.0) GO TO 400                                   BAL21800
        KI=NOD(K,LLLQ)                                                  BAL21900
       DHE=DHE+NXI(I)*RW(KI)                                            BAL22000
       DHN=DHN+NET(I)*RW(KI)                                            BAL22100
500      CONTINUE                                                       BAL22200
      DHX=JAC(1,1)*DHE+JAC(1,2)*DHN                                     BAL22300
      DHY=JAC(2,1)*DHE+JAC(2,2)*DHN                                     BAL22400
       VX=DHX*WX(LLLQ)                                                  BAL22500
       VY=DHY*WY(LLLQ)                                                  BAL22600
                RETURN                                                  BAL22700
C                                                                       BAL22800
C **********************************************************************BAL22900
C                                                                       BAL23000
C                                                                       BAL23100
                ENTRY VCENT(LLLQ.VXQ,VYA)                               BAL23200
C   THE AREA TO BE USED IN THE FORMULA Q=KIA IS COMPUTED                BAL23300
      ARX=(YL(3)+YL(4)-YL(1)-YL(2))/2                                   BAL23400
      ARX=ABS(ARX)                                                      BAL23500
      ARY=(XL(2)+XL(3)-XL(1)-XL(4))                                     BAL23600
      ARY=ABS(ARY)/2                                                    BAL23700
                VXQ=FLOWX(LLLQ)/ARX                                     BAL23800
                VYA=FLOWY(LLLQ)/ARY                                     BAL23900
                RETURN                                                  BAL24000
C**************************************************************         BAL24100
C                                                                       BAL24200
C                                                                       BAL24300
       SUBROUTINE FLOWW(LLL)                                            BAL24400
C                                                                       BAL24500
       DHE=0.0                                                          BAL24600
       DHN=0.0                                                          BAL24700
       M4=NODE(LLL)                                                     BAL24800
```

```
        CALL  DERIVE(LLL,M4)                                            BAL24900
       K=0                                                              BAL25000
       DO 7 I=1,M4                                                      BAL25100
4      K=K+1                                                            BAL25200
       IF(NOD(K,LLL).EQ.0) GO TO 4                                      BAL25300
       KI=NOD(K,LLL)                                                    BAL25400
       DHE=DHE+NXI(I)*R(KI)                                             BAL25500
       DHN=DHN+NET(I)*R(KI)                                             BAL25600
7      CONTINUE                                                         BAL25700
      DHX=JAC(1,1)*DHE+JAC(1,2)*DHN                                     BAL25800
      DHY=JAC(2,1)*DHE+JAC(2,2)*DHN                                     BAL25900
C   THE AREA TO BE USED IN THE FORMULA Q=KIA IS COMPUTED                BAL26000
      ARX=(YL(3)+YL(4)-YL(1)-YL(2))/2                                   BAL26100
      ARX=ABS(ARX)                                                      BAL26200
      ARY=(XL(2)+XL(3)-XL(1)-XL(4))                                     BAL26300
      ARY=ABS(ARY)/2                                                    BAL26400
       ARXX=ARX                                                         BAL26500
       ARX=DETJAC*4/ARY                                                 BAL26600
       ARY=DETJAC*4/ARXX                                                BAL26700
C                                                                       BAL26800
C                                                                       BAL26900
       RETURN                                                           BAL27000
C                                                                       BAL27100
C******************************************************************     BAL27200
      END                                                               BAL27300
```

```
C THIS ROUTINE IS USED TO PRINT FLOWS AND FLUXES                       FLO  100
C                                                                      FLO  200
      SUBROUTINE FLOWS(LM,LN,R,R1,FLOWX,FLOWY)                         FLO  300
      DIMENSION R(LN),R1(LN),FLOWX(LM),FLOWY(LM)                       FLO  400
      COMMON/HI/TITLE(25),V(26),VV(26)                                 FLO  500
C                                                                      FLO  600
C*********************************************                         FLO  700
      RETURN                                                           FLO  800
      ENTRY FFLOW                                                      FLO  900
      WRITE(14,V) (R1(I),I=1,LN)                                       FLO 1000
      RETURN                                                           FLO 1100
      ENTRY FFFLOW                                                     FLO 1200
      WRITE(14,V) (R(I),I=1,LN)                                        FLO 1300
      RETURN                                                           FLO 1400
C                                                                      FLO 1500
      ENTRY WFLOW(S)                                                   FLO 1600
      PRINT 1,S                                                        FLO 1700
 1    FORMAT(1X//1X,'TIME STEP  ',G12.6/1X,'FLOWS IN THE X DIRECTION') FLO 1800
       PRINT VV,(FLOWX(I),I=1,LM)                                      FLO 1900
      PRINT 2                                                          FLO 2000
  2    FORMAT(1X/1X,'FLOWS IN THE Y DIRECTION')                        FLO 2100
      PRINT VV,(FLOWY(I),I=1,LM)                                       FLO 2200
      RETURN                                                           FLO 2300
C                                                                      FLO 2400
C*********************************************                         FLO 2500
C                                                                      FLO 2600
      ENTRY HPRINT(S)                                                  FLO 2700
      PRINT 10,S                                                       FLO 2800
 10    FORMAT(1X//1X,'TEMPERATURE DISTRIBUTION AT TIME STEP  ',G12.6)  FLO 2900
      PRINT V,(R(I),I=1,LN)                                            FLO 3000
      RETURN                                                           FLO 3100
C                                                                      FLO 3200
C************************************************                      FLO 3300
C                                                                      FLO 3400
       ENTRY WPRINT(S)                                                 FLO 3500
      PRINT 100,S                                                      FLO 3600
100       FORMAT(1X//1X,'POTENTIAL DISTRIBUTION AT TIME STEP  ',G12.6) FLO 3700
      PRINT V,(R1(I),I=1,LN)                                           FLO 3800
      RETURN                                                           FLO 3900
      END                                                              FLO 4000
```

```
C**********************************************************************
      SUBROUTINE PARAM(LM,LN,BBB,XLOC,YLOC,NOD,PCX,DIFF,R,R1,WX,WY,
C
C    THIS SUBROUTINE IS USED TO COMPUTE THE DISPERSION COEFFICEIENTS,
C     THE LOCALIZED COORDINATES, AND  CHANGES IN HYDRAULIC
C    CONDUCTIVITY WITH TEMPERATURE
C
C**********************************************************************
     * A,B,BOT,KAREAL)
      INTEGER BBB
      DIMENSION XLOC(LN),YLOC(LN),NOD(12,LM),PCX(LM),DIFF(LM,2),R(LN)
     *, R1(LN),WX(LM),WY(LM),A(LM),B(LM),BOT(BBB)
      REAL JAC
      COMMON/AAA/XL(4),YL(4),DETJAC,JAC(4,4)/ACC/KK(12),K1,K2,K3,K4
     */HM/PXX,PYY,PXY,KAD/AM/NLA,PCW
C
      RETURN
C
C*******************************************************
C THE ROUTINE THAT CALCULATES THE DISPERSION COEFFICIENTS
C
      ENTRY MECD(LLL,VX,VY)
        V=VX*VX+VY*VY
       IF(V.LE.0) RETURN
       V=SQRT(V)
      DL=DIFF(LLL,1)*V
      DT=DIFF(LLL,2)*V
       PXX=(DL*VX*VX/V/V+DT*VY*VY/V/V)*PCW
      PYY=(DT*VX*VX/V/V+DL*VY*VY/V/V)*PCW
      PXY=((DL-DT)*VX*VY/V/V)*PCW
      PXY=ABS(PXY)
      RETURN
C
C*******************************************************
C
       ENTRY CORD(LLLQ)
      COMMON/AF/II,JJ,L1(4)
C  THIS ROUTINE COMPUTES LOCALIZED COORDINATES
      K1=NOD(1,LLLQ)
      K2=NOD(2,LLLQ)
      K3=NOD(3,LLLQ)
      K4=NOD(4,LLLQ)
      AQ=XLOC(K1)
      BB=XLOC(K2)
      C=XLOC(K3)
      D=XLOC(K4)
      E=AMIN1(AQ,BB,C,D)
      XL(1)=AQ-E
      XL(2)=BB-E
      XL(3)=C-E
      XL(4)=D-E
      AQ=YLOC(K1)
      BB=YLOC(K2)
      C=YLOC(K3)
      D=YLOC(K4)
      E=AMIN1(AQ,BB,C,D)
      YL(1)=AQ-E
      YL(2)=BB-E
      YL(3)=C-E
      YL(4)=D-E
C
C
      RETURN
```

```
C
C
C*************************************************************
C
      ENTRY PE(MS)
C
      IF(MS.EQ.0) RETURN
     DO 1 J=1,LM
     N1=NOD(1,J)
     N2=NOD(2,J)
     N3=NOD(3,J)
     N4=NOD(4,J)
     T=(R(N1)+R(N2)+R(N3)+R(N4))/4
C
C   THE VISCOSITY RELATIONSHIP
C
     U=1.917-.05635*T+.00071*T*T
     WX(J)=1.21/U*A(J)
     WY(J)=1.21/U*B(J)
1    CONTINUE
     RETURN
      ENTRY PEE(BBQQ,JQQ)
      IF(KAREAL.EQ.0) RETURN
      N1=NOD(1,JQQ)
      N2=NOD(2,JQQ)
      N3=NOD(3,JQQ)
      N4=NOD(4,JQQ)
C    GEOMETRIC MEANS ARE CALCULATED FOR AQUIFER DEPTH
      BBQQ=(R1(N1)-BOT(N1))*(R1(N2)-BOT(N2))*(R1(N3)-BOT(N3))*(R1(N4)-B
     *OT(N4))
      BBQQ=ABS(BBQQ)
      BBQQ=BBQQ**0.25
      RETURN
      END
```

```
C*********************************************************************
C   THIS ROUTINE IS USED TO CHANGE BOUNDARY CONDITIONS OR PARAMETERS
C     AT EACH TIME STEP     FIVE ENTRY POINTS ARE PROVIDED
C
       SUBROUTINE BOUNDA(LM,LN,Z,R,R1,HEAD,HEA,FLOWX,FLOWY,HEAT,INFLOW
     *,NWATER,AWATER,NHEAT,AHEAT,NCON,TINF,CONV,ALOC,NEL,AEL,CBAL
     *,LEL,LHEAT,LWATER,PCW,AY,AZ,LINEW,LINEH,NLINEW,NLINEH,
     * ALINEW,ALINEH)
       INTEGER Z,AY(4),AZ(4)
       REAL INFLOW,N11
       COMMON/ATHICK/ASIZE,NTHICK,THICK,ERROR/BM/RY(4800,3)
     */BOUND/DIST,N1,N2,N3,N4,N5,N6,N7,N8,N9,N10
     */CONT/LA,LB,LC,LD,LE,LF,LG,LH/CON/MC1,MC2,NCONV/ME/MEQ
C--LINKING INFORMATION
        DIMENSION FLOWX(LM),FLOWY(LM)
C--INFORMATION FOR POINT SOURCES
       DIMENSION NWATER(Z),AWATER(Z),NHEAT(Z),AHEAT(Z)
C  RECHARGE RATE INFORMATION
      *, HEAT(LM),INFLOW(LM),NLINEW(Z,2),NLINEH(Z,2),ALINEW(Z),ALINEH(Z)
C--INFORMATION ON CONVECTIVE BOUNDARIES
     * ,NCON(Z,3),TINF(Z),CONV(Z),ALOC(Z),NEL(Z,2),AEL(Z,2),CBAL(Z,2)
C--BOUNDARY CONDITIONS
      DIMENSION HEAD(LN),HEA(LN)
C--INITIAL CONDITIONS AND ANSWERS
       DIMENSION R(LN),R1(LN)
C
       RETURN
C
C***************************************************************
C
       ENTRY LAKE
      READ, DIST,N1,N2,N3,N4,N5,N6,N7,N8,N9,N10,N11
      READ,M,NUMBER
      DO 110 K=1,M
 110   READ,A,B,C
      MM=NUMBER-M
      DO 111 K=1,MM
 111   READ,RY(K,3),RY(K,1),RY(K,2)
      RETURN
C
C***************************************************************
C
C
       ENTRY BVAL(KSQ,ALPHA)
       K=KSQ*ALPHA
       T1=RY(K,1)-5.0
       DO 100 J=1,LEL
100      AEL(J,2)=T1
       RETURN
C
C***************************************************************
C
       ENTRY BOUND(KS)
C
             K=KS*N10+8
      T1=RY(K,1)
      T2=RY(K,2)
      AA=ALOG(T1)
      AB=(ALOG(T2)-AA)/54
      T3=AA+DIST*AB
      T3=EXP(T3)
       DO 59 J=N1,N2,N3
59     HEAD(J)=T3
      DO  60 J=N4,N5,N6
 60     HEAD(J)=T3-2.5                                                  BOU 6500
       DO 200  J=1,LEL                                                  BOU 6600
       TINF(J)=T3                                                       BOU 6700
200       AEL(J,2)=T3                                                   BOU 6800
      IF(NCONV.LT.1) GO TO 300                                          BOU 6900
       LEA=LEL+1                                                        BOU 7000
      DO 500 J=LEA,NCONV                                                BOU 7100
       TINF(J)=RY(K-2,1)-N11                                            BOU 7200
      IF(RY(K-2,3).LT.50) TINF(J)=RY(K-2,3)-N11                         BOU 7300
 500    CONTINUE                                                        BOU 7400
       PRINT 555, (NCON(J,1),TINF(J),J=1,NCONV)                         BOU 7500
555    FORMAT(1X,'NODE # AND TEMP AT INF.',8(2X,I3,2X,F5.2))            BOU 7600
      GO TO 400                                                         BOU 7700
 300     CONTINUE                                                       BOU 7800
      DO 700  J=N7,N8,N9                                                BOU 7900
 700      HEAD(J)=T3-N11                                                BOU 8000
 400     CONTINUE                                                       BOU 8100
      RETURN                                                            BOU 8200
C                                                                       BOU 8300
C****************************************************                   BOU 8400
C                                                                       BOU 8500
C                                                                       BOU 8600
       ENTRY CHANG                                                      BOU 8700
C                                                                       BOU 8800
       RR=R(130)+R(131)+R(145)+R(146)                                   BOU 8900
       RR=RR/4                                                          BOU 9000
        DO 10 J=1,LHEAT                                                 BOU 9100
       N=NHEAT(J)                                                       BOU 9200
10     AHEAT(J)=R(N)*PCW*AWATER(J)                                      BOU 9300
       RETURN                                                           BOU 9400
C                                                                       BOU 9500
       ENTRY CHAN                                                       BOU 9600
       RETURN                                                           BOU 9700
       END                                                              BOU 9800
       SUBROUTINE EIGEN(LN,MBAND.MP,KSYM,CFACT,S,T.RI,HEAD)
      DOUBLE PRECISION S,T
      DIMENSION HEAD(LN), RI(LN), S(LN MBAND),T(LN,MBAND)
       COMMON/HI/TITLE(25),V(26),VV(26)
       PRINT 10
      COMMON/HH/LFLOW,LON
C
      C=1000
      DO 1 J=1,LN
      A=0
       IF(KSYM.GT.0) GO TO 6
       IF(HEAD(J).EQ 0) A=S(J,1)
      DO 2 K=2,MBAND
       IF(HEAD(J).NE.0) GO TO 1
       E=S(J,K)
2      A=ABS(E)+A
      MBA=MBAND-1
      DO 4 K=1,MBA
      NO=J-K
      IF(NO.LE.0) GO TO 5
       E=T(NO K+1)
4      A=ABS(E)+A
    5 CONTINUE
      B=T(J,1)*.5/A
       GO TO 8
6      DO 7 K=1,MP
       IF(S(J,K).GT.CFACT) GO TO 1
       E=S(J,K)
       E=ABS(E)
       IF(K.EQ.MBAND) E=S(J,K)
7      A=E+A
       B=T(J,MBAND)*0.5/A
8      CONTINUE
       RI(J)=B
       IF(B.LT.C) LQ=J
      C=AMIN1(B,C)
    1 CONTINUE
       PRINT V,(RI(J),J=1,LN)
      PRINT 11,C,LQ
       STOP
10     FORMAT(1X/1X,'STABILITY OF SOLUTION USING CRANK NICOLSON METHOD'
     *,4X/3X,'MAXIMUM TIME STEP FOR EACH NODE')/
11     FORMAT(1X/1X,'THE CRITICAL TIME STEP IS ',G12.6,'
     *CONSTRAINED AT NODE'.I5)
      END
```

```
C   SUBROUTINE THAT ALLOWS FOR A MOVING BOUNDARY IN A VERTICAL CROSS SEC
C
C
C
      SUBROUTINE ADJUST(NJUST,LM,LN,Z,HEAD,XLOC,YLOC,NODE,FLOWX,FLOWY,R
     *)
      DIMENSION NMOV(40,2), FLOWX(LM),FLOWY(LM),A(2),B(2),R(LN)
      DIMENSION BMOV(40,6),HEAD(LN),XLOC(LN),YLOC(LN),NODE(12,LM)
      READ,NSTEP,NPRINT,NPR,ERROR,FACTOR
       READ,(NMOV(J,1),NMOV(J,2),BMOV(J,3),BMOV(J,6),J=1,NJUST)
      KLT=0
      KSS=1
      RETURN
C
C*****************************************************
C
      ENTRY ADJUS(*,*)
      IF(KSS.EQ.2) GO TO 5
      DO 4 J=1,NJUST
      LL=NMOV(J,1)
      LL=ABS(LL)
      IF(LL.NE.0) GO TO 2
      LL=NMOV(J,2)
      LL=ABS(LL)
      L=NODE(4,LL)
      GO TO 3
2     CONTINUE
      L=NODE(3,LL)
3     CONTINUE
      BMOV(J,5)=YLOC(L)
4     CONTINUE
5     CONTINUE
      KLT=KLT+1
      READ,BKS,TIME,FACT
      DO 6 J=1,NJUST
6     BMOV(J,4)=BMOV(J,6)*FACT
      DO 8 J=1,NJUST
8     BMOV(J,4)=BMOV(J,4)/FACTOR
      SLENG=0.00001
      FLOW=0
      DO 40 J=1,NJUST
      DO 10 K=1,2
      NS=1
      LLL=NMOV(J,K)
      IF(LLL.LT.0) NS=2
      LLL=ABS(LLL)
      IF(LLL.EQ.0) GO TO 10
      CALL CORD(LLL)
      CALL VCENT(LLL,VX,VY)
      LA=NODE(3,LLL)
      LB=NODE(4,LLL)
      AA=(XLOC(LA)-XLOC(LB))
      A(K)=ABS(AA)
      B(K)=VY
10    CONTINUE
      LC=NMOV(J,1)
      LC=ABS(LC)
      IF(LC.EQ.0) A(1)=A(2)
      IF(LC.EQ.0) B(1)=B(2)
      IF(LLL.EQ.0) A(2)=A(1)
      IF(LLL.EQ.0) B(2)=B(1)
      IF(LLL.EQ.0) LB=NODE(3,LC)
      IF(NS.EQ.2) GO TO 20
      C=(B(1)*A(1)+B(2)*A(2))/(A(1)/2+A(2)/2)
      YLOC(LB)=YLOC(LB)+((-C+BMOV(J,4))*TIME/BMOV(J,3))
      HEAD(LB)=YLOC(LB)
      GO TO 40
 20   CONTINUE
      FLOW=B(1)*A(1)/2+B(2)*A(2)/2+FLOW
      SLENG=SLENG+A(1)/2+A(2)/2
 40   CONTINUE
C
C
C LAKE LEVEL LEVELLER
C
      HE=FLOW/SLENG
      DO 50 J=1,NJUST
      IF(NMOV(J,2).GT.-0.0001) GO TO 50
      LLL=NMOV(J,2)
      LLL=ABS(LLL)
      IF(LLL.EQ.0) GO TO 45
      MD=NODE(4,LLL)
      GO TO 47
 45   LLL=NMOV(J,1)
      LLL=ABS(LLL)
      MD=NODE(3,LLL)
 47   CONTINUE
      YLOC(MD)=(-HE+BMOV(J,4))*TIME/BMOV(J,3)+YLOC(MD)
      HEAD(MD)=YLOC(MD)
 50   CONTINUE
C
C   PRINTING ROUTINE
C
      LP=KLT/NPRINT
      LQ=KLT/NPR
      LP=LP*NPRINT
      LQ=LQ*NPR
      IF(KLT.EQ.LP) CALL WPRINT(BKS)
      IF(KLT.EQ.LQ) CALL WFLOW(BKS)
      IF(KLT.EQ.LQ) CALL BPRINT
C
      IF(KLT.GT.NSTEP) STOP
      KSS=2
      AC=0
      DO 70 J=1,NJUST
      LL=NMOV(J,1)
      LL=ABS(LL)
      IF(LL.NE.0) GO TO 60
      LL=NMOV(J,2)
      LL=ABS(LL)
      L=NODE(4,LL)
      GO TO 65
 60   CONTINUE
      L=NODE(3,LL)
 65   CONTINUE
       AD=BMOV(J,5)-YLOC(L)
      AD=ABS(AD)
      AC=AMAX1(AD,AC)
 70   CONTINUE
      IF(AC.GT.ERROR) KSS=1
      IF(KSS.EQ.2) RETURN 2
      PRINT 110,BKS,KLT
110   FORMAT(1X/1X,'STUCTURE MATRIX RECOMPUTED AT TIME STEP  ',G8.3,
     *'   ITERATION   ',I5)
      RETURN 1
      END
```

APPENDIX D

ESTIMATION OF AQUIFER PARAMETERS BY USING SUBSURFACE TEMPERATURE DATA

Flowing ground water distorts the normal distribution of subsurface temperatures. Researchers interested in determining the geothermal heat flux have long been aware of the distorting influence moving ground water can have on the measured geothermal gradient (Kirge 1939). Only a few ground-water researchers, however, have attempted to use subsurface temperature information to calculate the rate and direction of ground-water flow.

Stallman (1960) aroused interest in the use of subsurface temperature as an indirect manifestation of ground-water velocity with a presentation of a differential equation describing heat transport in the subsurface and with the suggestion that temperature measurements might be a useful means for indirectly determining aquifer characteristics. An analytical solution was developed by Stallman (1965) for determining ground-water velocities in a homogeneous medium when the boundary conditions of heat and water movement are, respectively, (1) a sinusoidal temperature fluctuation of constant amplitude at the land surface, and (2) a constant and uniform percolation rate normal to the land surface. Bredehoeft and Papadopulos (1965) developed an analytical solution for determining the ground-water velocity from subsurface temperature data in an isotropic, homogeneous, and fully saturated semiconfining layer in which all flow is vertical.

The analytical solution developed by Stallman (1965) has not been used extensively. Its main drawbacks are the assumptions that the water table is at the surface and that all flow is vertical. Taking a different approach, Nightingale (1975) estimated vertical recharge rates from an infiltration pond with a sinusoidal temperature distribution, but he assumed that conductive transfer processes were negligible and that velocity could be calculated by using only the lag between surface temperatures and temperatures at a point in the subsurface.

Several researchers have used Bredehoeft and Papadopulos's analytical solution. Cartwright (1970) successfully used temperature anomalies to estimate the amount of vertical movement of ground water in the Illinois Basin. He matched the average temperature profile to the $\beta=-1$ curve of Bredehoeft and Papadopulos (1965) to obtain a discharge rate of 1.52 cm/yr from the deep aquifer, a value that agrees with estimates of ground-water discharge into streams in the basin. Sorey (1971) used the Bredehoeft and Papadopulos technique to estimate the rate of upward movement through semiconfining beds in the San Luis valley of Colorado and the Roswell basin of New Mexico. His conclusions suggest that pumping tests and water-budget methods are often preferable because of limitations imposed by instabilities in the borehole fluids and the measurement detail required. Boyle and Saleem (1978) used the technique to estimate vertical flow rates through a clay-rich glacial drift semiconfining layer in the Chicago area. They obtained good agreement with values calculated by using the water-budget method.

The available analytical techniques only provide solutions for a very small group of problems in which flow is in the vertical direction and boundary conditions are specialized. In most aquifers the dominant direction of water movement is horizontal. Noticeable temperature variations have been observed in the horizontal direction in ground-water systems (Winslow 1962, Schneider 1962 and 1964, Mink 1964, Parsons 1970, Supkow 1971, Cartwright 1973). The researchers all implicitly assumed that the variations in temperature were caused by ground-water flow, but none were able to quantify flows by using this information.

In the course of the research reported here an attempt was made to use the measured horizontal and vertical distribution of subsurface temperatures to estimate the rate of flow from a cooling lake situated on an alluvial aquifer. A numerical technique was used by which ground-water velocities and hydraulic conductivities in a two-dimensional system with nonuniform boundary conditions can be determined by a trial-and-errror procedure from subsurface temperature information. The procedure is not recommended for use in routine ground-water flow system analysis since traditional methods for defining flow systems are simpler to use and more reliable.

MATHEMATICAL MODEL

Given the following assumptions: (1) thermal equilibrium between the liquid and soil particles is achieved simultaneously, (2) the density of the soil particles is constant, (3) the heat capacity is constant, and (4) the system is chemically inert, then the general differential equation for simultaneous heat and fluid flow in a two-dimensional aquifer is

$$\frac{\partial}{\partial x_i}\left(D_{ij}\frac{\partial T}{\partial x_j}\right) - \rho C_w \frac{\partial (q_i T)}{\partial x_i} - \rho C_s \frac{\partial T}{\partial t} + R = 0 \;, \quad i,j, = 1,2 \;, \qquad \text{(D-1)}$$

where T = temperature, T; D_{ij} = coefficient of dispersion, H/tTL; ρC_w = heat capacity of water, H/L^3T; ρC_s = heat capacity of the saturated media, H/L^3T; q_i = specific discharge or ground-water velocity, L/t; R = rate of heat injection or discharge, H/L^3t; and x_1, s_2 = cartesian coordinates L.

The goal is to solve Eq. (D-1) for the velocity distribution; then hydraulic conductivities can be determined from

$$K_{ij} = -q_i \left(\frac{\partial \phi}{\partial x_j}\right)^{-1} \qquad i,j=1,2 \;; \qquad \text{(D-2)}$$

where K_{ij} = hydraulic conductivity tensor, L/t; and ϕ = head, L.

Velocities in the aquifer can be determined from Eq. (D-1) either directly or indirectly according to whether velocities are obtained directly by using the temperature distribution as a known in the differential equation, or whether velocities are obtained as a solution to a nonlinear optimization problem in which a set of calculated temperatures is matched to an observed set. Direct methods require that the temperature distribution be known completely and that it closely represent the true solution to the differential equations (Neuman 1973). These types of solutions to the inverse problem are currently a subject of active research and were not used in the present study.

The equation describing the two-dimensional flow of water through a nonhomogeneous aquifer in steady state may be written as

$$\frac{\partial}{\partial x_i}\left(bK_{ij}\frac{\partial\phi}{\partial x_j}\right) + W = 0 \ , \quad i,j=1,2 \ , \tag{D-3}$$

where W = water recharge rate per unit area, L/t; and b = thickness, L.

The ground-water velocity or specific discharge can then be determined from

$$q_i = -K_{ij}\frac{\partial\phi}{\partial x_j} \qquad i,j=1,2 \ . \tag{D-4}$$

Acquisition of some information on subsurface permeability is desirable for initial estimates of velocities by using the general differential equation describing water flow in an aquifer. Hydraulic conductivities can then be adjusted until the computed velocity distribution, when input into Eq. (D-1), produces a temperature distribution that is reasonably close to the observed temperature data.

The parameter estimation problem could be viewed as a classical nonlinear regression problem in which a solution to the linked differential equations [Eq. (D-1), Eq. (D-3), and Eq. (D-4)] forms the regression equation and in which all unknown quantities are parameters (Cooley 1977). The problem was not viewed in this manner because (1) sufficient temperature data will generally not be available to insure a well-conditioned solution, and (2) Eq. (D-1) behaves as a hyperbolic paraboloid when convective transport dominates, which means that small errors in the input parameters can cause large errors in the output.

Instead, a trial-and-error procedure was used to estimate hydraulic conductivities. This technique is conceptually simple, but has several drawbacks: (1) It can be expensive, (2) it is very time consuming, (3) an answer cannot always be obtained, and (4) if an answer is obtinaed, it is probably not the best estimate, and its relation to the best estimate is unknown.

Several difficulties are encountered when Eq. (D-1) and Eq. (D-3) are linked and solved in a trial-and-error procedure to estimate hydraulic conductivities. The best documented difficulty is that, when hydraulic conductivities are known, researchers have not been very successful in solving for a known conservative contaminant distribution, even though the conservative mass transfer problem is mathematically simpler than the heat transport problem. The differential equations are of the same form, but the conservative mass transport problem has two fewer parameters because of the common assumption that molecular diffusion, which is equivalent to thermal conductivity, is negligible.

Attempts to simulate a known conservative contaminant distribution in an aquifer with a deterministic model are not numerous. Pinder (1973) simulated the observed chromium contamination on Long Island; Bredehoeft and Pinder (1973) simulated the known chloride distribution in the Brunswick aquifer; Konikow (1976) simulated the chloride distribution in the vicinity of the Rocky Mountain Arsenal; Robertson (1974) simulated chlorides and other contaminants in the vicinity of the Idaho Test Site; and Robson (1978) simulated total dissolved solids in a shallow alluvial aquifer near Barstow, Calif. In all these simulations velocities within the aquifer were determined by using equations similar to Eq. (D-3) and Eq. (D-4). In these equations hydraulic conductivities were adjusted by a trial-and-error procedure until predicted aquifer heads closely matched observed aquifer heads. An equation similar to Eq. (D-1) was used to simulate transport of contaminants, and the porosity and dispersivity parameters were then adjusted until the observed chemical concentration patterns were matched. Even when hydraulic conductivities are known, the chemical concentration distribution could not be simulated without a trial-and-error adjustment procedure. Also, even after the researchers had obtained what they considered to be a best fit between the simulated and the observed data, the fit in all cases was less than perfect. Anderson (1979) discusses some of the problems involved in applying contaminant transport models.

Since the temperature data will generally be sparse, the trial-and-error procedure of parameter estimation for the linked Eq. (D1) and (D-3) is generally non-tractable unless several of the parameters are constrained. In this analysis, in which hydraulic conductivities in a steady-state water-flow problem were estimated, thermal conductivities, heat capacities, and dispersivities were fixed. The rationale for fixing these parameters was that the thermal conductivity and the heat capacity of most saturated glacial materials fall within a small range, and they can be measured accurately in the laboratory. Dispersivities were assumed to be small because element sizes were chosen so that intra-element inhomogeneities were minimized.

These procedures left only hydraulic conductivities to be adjusted. Generally, even with hydraulic conductivities as the only unknowns, problems were ill conditioned unless the hydraulic conductivities were constrained to be within a small range. Therefore, this technique is only useful for refining estimates of hydraulic conductivity that are determined with other procedures.

The following procedure was used to refine estimates of the hydraulic conductivity distribution at the Columbia Generating Station site by using subsurface temperature data:

1) Hydraulic conductivity distributions were proposed on the basis of stratigraphic information and field tests, and these distributions were

then tested using Eq. (D-3) to determine if the known potential distribution could be predicted within a set error criterion.

2) Once the potential distribution could be predicted, the relative magnitude of the hydraulic conductivities was adjusted in a model linking Eq. (D-1) and Eq. (D-3) to determine which factor gave the best fit to the observed temperature data.

3) If the best fit of the temperature data obtained in step 2 was not good, steps 1 and 2 were repeated. If the fit was judged to be acceptable, thermal conductivities and dispersivities were changed to determine the sensitivity of the estimate to changes in these parameters.

SOLUTION PROCEDURE

The finite element method was used to solve Eq. (D-1) and Eq. (D-3) for temperatures in an aquifer subject to the boundary conditions described in section 5 and appendix B. Each cross section modeled was divided into 100-150 quadrilateral elements. The procedure used to link the equations was: (1) Eq. (D-3) was solved for head at each node and Eq. (D-4) was solved for velocity; (2) Eq. (D-1) was solved for temperature at each node; (3) the solution to Eq. (D-1) was stepped forward in time by using the Crank-Nicolson approximation for the time derivative for a specified number of time steps; (4) the hydraulic conductivities, which are a function of temperature, were adjusted for the new temperature distribution; and (5) steps (1) to (4) were repeated.

FIELD DATA

Temperatures were recorded weekly at 40-110 points in the subsurface in the vicinity of the Columbia Generating Station (Figure A-1) from August 1976 to January 1978. The monitoring techniques and the locations of the data points are described in appendix A. The data collected on 7 October 1977 are presented in a fence diagram in Figure (D-1). The data collected in cross-section A-A' and B-B' of Figure 6 are presented in section 5. Temperatures recorded in an array of wells on the east side of the cooling lake are shown in Figure (D-2).

RESULTS

The temperature data from cross-sections A-A' and B-B' were used to refine estimates of hydraulic conductivity and to refine previous estimates of flow in these cross sections from the cooling lake to the wetland west of the lake. Andrews (1976) modeled flow in these cross sections and obtained average annual flow rates of 4.5 m^2/day (m^3/day per meter width of the dike and 3.5 m^2/day, respectively. By using the trial-and-error procedure described in this appendix, values of 5.2 m^2/day and 4.3 m^2/day were obtained for flow in these two cross sections, respectively. An error criterion of 0.05 m was used in step 1. Thermal conductivities were set equal to values obtained in the laboratory. Longitudinal dispersivity was set to 20 cm, and transverse dispersivity was set to 5 cm on the basis of intro-element homogeneity considerations.

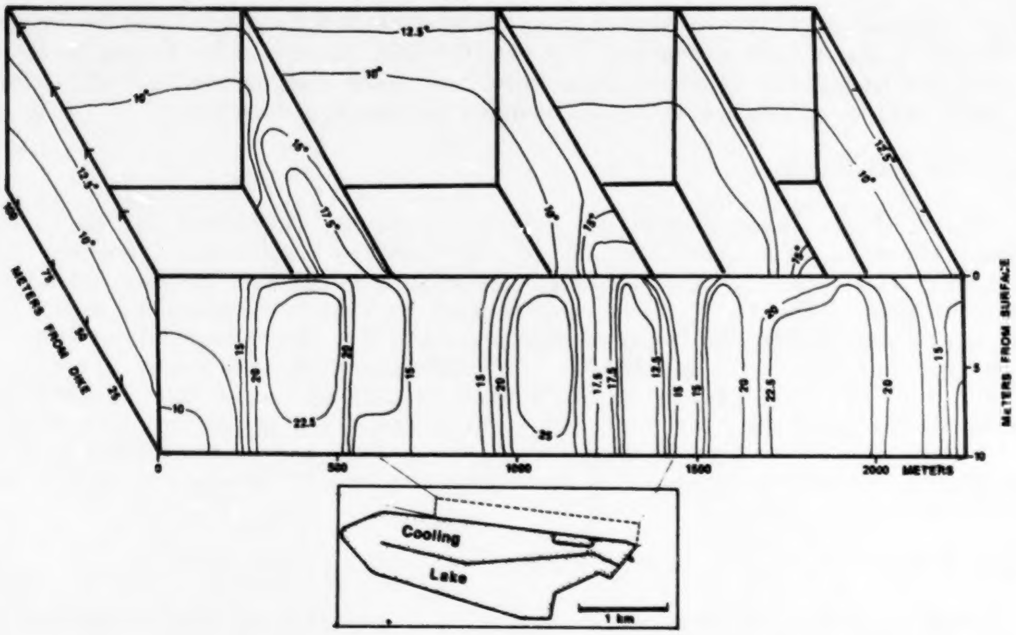
Temperatures in the Marsh on October 7, 1977

Figure D-1. Temperatures (°C) in a section of the marsh adjacent to the cooling lake at the Columbia Generating Station site on 7 October 1977. The three-dimensional diagram depicts temperatures in a 2,500-m section along the dike which extends outward from the dike for a distance of 110 m and to a depth of 10 m.

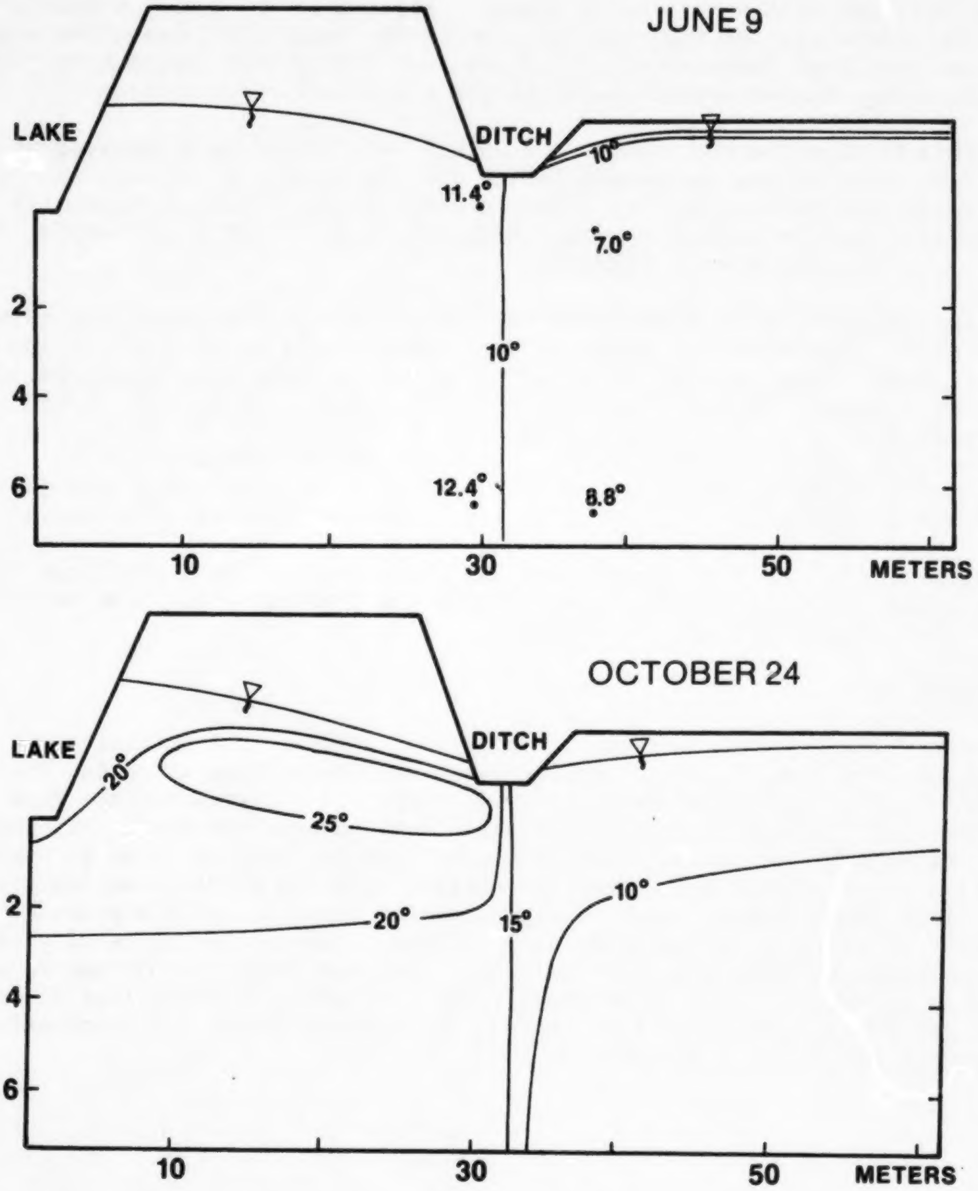
JUNE 9
LAKE
DITCH
10°
11.4°
7.0°
2
10°
4
6
12.4°
8.8°
10
30
50
METERS
OCTOBER 24
LAKE
DITCH
20°
25°
10°
2
20°
15°
4
6
10
30
50
METERS

Figure D-2. Temperatures in a 60-m portion of cross-section B-B' of Figure 6 adjacent to the drainage ditch east of the cooling lake on 9 June 1977 and 24 October 1977.

The hydraulic conductivity distribution that best reproduced the observed temperature data was not sensitive to changes in the thermal conductivities or dispersivities within reasonable limits. Adjusting the thermal conductivities and the dispersivities did, however, reduce the residuals between the observed and the simulated temperatures. The changes in simulated temperature that resulted when dispersivities were changed are described in section 5.

This trial-and-error procedure does not lend itself to estimation of the standard error of the parameters in the best fit mode. It is only an intuitive observation that the standard error of the estimated hydraulic conductivities are reduced by using this tehcnique rather than stopping at step 1 in the parameter estimation.

The temperature data were also used to attempt to determine flow rates in five other cross sections (Figure 15) at the Columbia Generating Station site. In all these cross sections only limited potential data were available, and several hydraulic conductivity distributions could be found that would reproduce the known potential distribution equally well. Since the temperature data were also limited, several hydraulic conductivity distributions which represented widely varying flow rates could reproduce the observed data equally well. Only in the simulated cross section located near the intake of the cooling lake were the results satisfactory. (Of the five cross sections, the best temperature and potential data were available for this cross section.) In this cross section the flow rates into the wetland were estimated to average 7.4 m^2/day.

CONCLUSIONS

Unfortunately, the information in Figure (D-1) was not sufficient to estimate flow rates from the cooling lake into the wetland along the dike. Figure (D-1) does illustrate vividly the complexity of ground-water flow patterns and hydraulic conductivity distributions near the dike. Although flow rates from the cooling lake to the wetland in the area shown in Figure (D-1) are most likely all within the range of 3.5-7.5 m^2/day, the temperature contrasts are dramatic. Small changes in peat thickness or the presence or absence of a clay lens can cause very different temperature patterns even though flows are approximately the same. Keys and Brown (1978) reached a similar conclusion. The temperature data, although indicating that flows are much greater in some areas than others, are not sufficient for determining hydraulic conductivity distributions.